LECTURE NOTES

B.TECH 1st YEAR

AY:(2024-25)

DEPARTMENT OF MECHANICAL ENGINEERING

ENGINEERING MECHANICS
[23ES1004]

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MODERN ENGINEERING & MANAGEMENT STUDIES

DEPARTMENT OF MECHANICAL ENGINEERING

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Module I (10 Hrs)

Concurrent forces on a plane: Composition, resolution and equilibrium of concurrent coplanar forces, method of moment. General case of forces on a plane: Composition and equilibrium of forces in a plane, plane trusses, method of joints and method of sections.

Module II (6 Hrs)

Friction: Fundamentals and Problems involving friction, Ladder, Wedges. Principle of virtual work.

Module III (8 Hrs)

Parallel forces on a plane: General case of parallel forces, center of parallel forces and center of gravity, Centroid of plane and composite figures, Theorems of Pappus and Guildins. Moment of inertia: Plane figure with respect to an axis in its plane and perpendicular to the plane, Polar moment of inertia, parallel axis theorem.

Module IV (8 Hrs)

Rectilinear translation: Kinematics, Principle of dynamics, D Alembert?s Principle, Principle of work and energy for a particle and a rigid body, Conservation of energy, Principle of impulse and momentum for a particle and a rigid body, Conservation of momentum, System of rigid bodies, Impact, direct and central impact, coefficient of restitution.

Module V (8 Hrs)

Curvilinear translation: Kinematics, Equation of motion, Projectile, D Alembert?s principle of curvilinear motion. Kinematics of rotation of rigid body.

MECHANICS

→ It is defined as that branch of science, which describes and predicts the conditions of rest or motion of bodies under the action of forces.

→ Engineering Mechanics applies the principle of mechanics to design, taking into account the effect of forces.

Unit Conversion

1 m = 100 cm = 1000 mm

1 km = 1000 m

1 cm^2 = 100 mm^2

1 m^2 = 10^6 mm^2

1 kgf = 9.81 N = 10 N

1 KN = 10^3 N

1 MPa = 1 N/mm^2

1 GPa = 10^3 N/mm^2

1 Pascal = 1 N/m^2

1 degree = $\frac{\pi}{180}$ radians

Quantity

Scalar Quantity

→ A Scalar Quantity is one which can be completely specified by its magnitude only.

→ • Length • Mass • Distance
• Density • Area • Temperature
• Volume • Speed • Time
• Energy • Work • Moment of Inertia

Vector Quantity

→ "A vector Quantity is one which requires magnitude & direction both to completely specified it".

→ • Displacement • Force
• Velocity • Angular displacement
• Angular Velocity • Momentum
• Moment • Impulse
• Weight
• Acceleration

Space :→ It is a region in all directions encompassing the universe. It is a geometric position occupied by bodies. These positions are describe by linear or angular measurements with reference to a defined system of co-ordinates.

Time :→ Time is a measurement to measure a duration between successive events.

→ In the study of statics time does not play important role.

→ In dynamics time is very important parameter.

Statics :→ Statics deal with the condition of equilibrium of bodies acted upon by forces.

Particle :→ A Particle is ideally dimensionless. But it has a very small mass.

Rigid Body :→ Nobody is perfectly rigid, however rigid body is defined as a body in which particle do not change their relative positions under the action of any force or torque.

→ Rigid body is ideal body. When the body does not deform under the action of A force or A torque, Body is said rigid.

Deformable Body :> When a body deforms due to a force or a torque it is said deformable body.

→ Material generates stresses against deformation.

Force :> Force is an agent, which generates or tends to generates and destroy or tends to destroy the motion in a body.

Characteristics of Force :>

→ It has a magnitude
→ It has a direction
→ It is a Vector quantity
→ It has a point of application
→ It has a nature – Tensile Force
– Compressive force
– Pull force
– Push force

When two or more forces act on a body, they are called to for a system of forces.

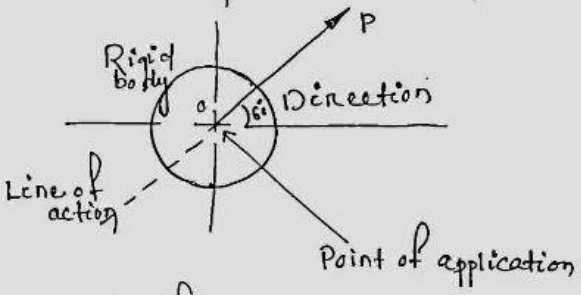
→ Unit = N, KN, Kg etc.

→ Magnitude – Magnitude of force indicates the amount of force (expressed as N or KN) that will be exerted on another body.

→ Direction - Angular position of a Force with some reference fixed axis.

→ Nature - The nature of force may be tensile or compressive.

→ Point of Application - The point at which the force acts on the body is called point of application.

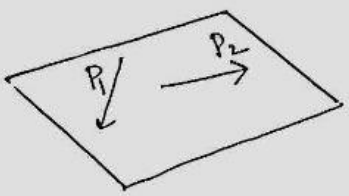


Types of Force

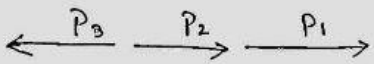
→ Contact Force
→ Body Force
→ Point force and distributed force
→ External force & internal force
→ Action & Reaction
→ Friction force
→ Wind force
→ Hydrostatic force
→ Cohesion & Adhesion
→ Thermal force

System of Forces

→ Coplanar Forces :→ The forces, whose lines of action lie on the same plane, are known as co-planar forces.

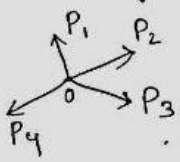


→ Collinear forces :→ The forces, whose lines of action lie on the same line, are known as collinear forces.

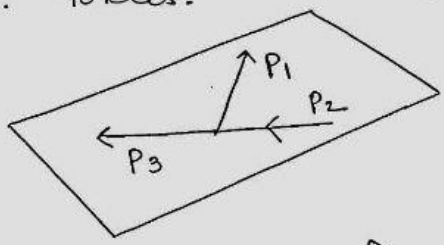


Concurrent Forces :→ The forces, which meet at one point, are known as concurrent forces.

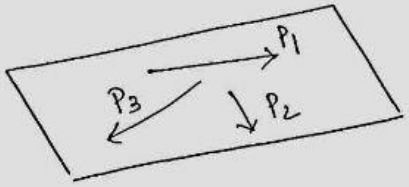
→ The concurrent forces may or may not be collinear.



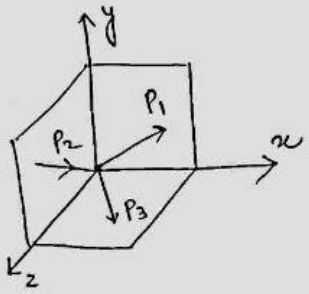
Coplanar Concurrent Forces :→ The forces, which meet at one point and their line of action also lay on the same plane, are known as coplanar concurrent forces.



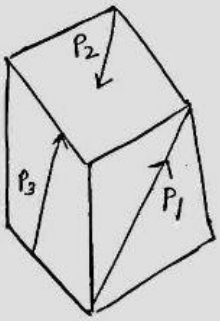
Coplanar non-concurrent forces :→ The forces, which do not meet at one point, but their lines of action lie on the same, are known as coplanar non-concurrent forces.



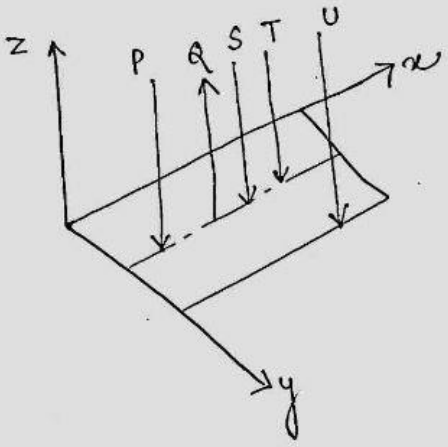
Non-Coplanar concurrent Forces :→ The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-concurrent forces.



Non-Coplanar non-concurrent Force :⇒ The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.



Parallel Forces :⇒



a. Like [P-S-T-U]
b. Unlike [All]
c. Coplanar [P-Q-S-T]
d. Non-Coplanar [All]

Principle of Individual Forces

Principle of Transmissibility :→

→ "If the force act at a point on a rigid body, it may also be considered to act at any other point on its line of action, provided the point is rigidly connected with the body".

Principle of Superposition of forces :→

" If two equal, opposite and collinear forces are added to or removed from the system of forces, there will be no change in the position of the body. This is known as principle of Superposition of Forces".

Coplanar Concurrent Forces

Resultant Force :→ If number of Forces acting simultaneously on a particle, it is possible to find out a single force which could replace them or produce the same effect as of all the given forces is called resultant force.

Methods of finding Resultant :→

1- Parallelogram Law of Forces (For 2 forces)

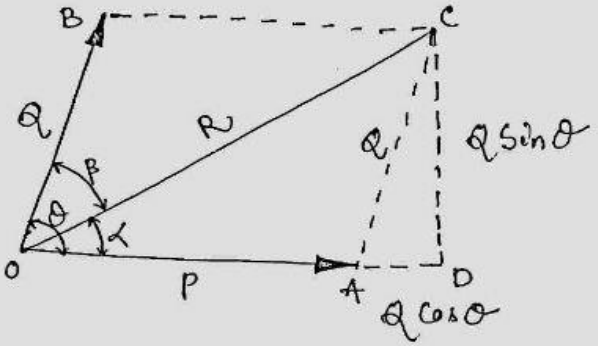
2- Triangle Law (For 2 Forces)

3- Lami's theorem (For 3 Forces)

4- Method of resolution (For more than 2 Forces)

1. **Parallelogram Law of Forces :⇒**

→ If two forces acting on a particle at an angle θ, be represented in magnitude & direction by the two sides of parallelogram, then the diagonal passing in between them represents the direction & magnitude of the resultant.



Mathematically,

$$\text{Magnitude}, \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\text{Direction}, \quad \tan\theta = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\tan\beta = \frac{P\sin\theta}{Q + P\cos\theta}$$

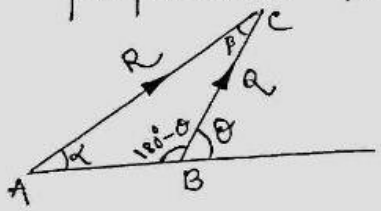
∴ Where, R = Resultant Force

θ = angle between P & Q

α = angle between P & R

β = angle between Q & R

According to the Triangle law, each side of a triangle is proportional to the sine angle b/w other two sides.



$$\frac{AB}{\sin\beta} = \frac{BC}{\sin\alpha} = \frac{AC}{\sin(180^\circ - \theta)}$$

$$\Rightarrow \frac{P}{\sin\beta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\theta}$$

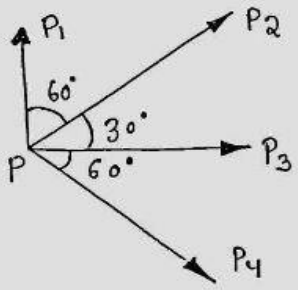
2. Triangle Law of Forces :⇒

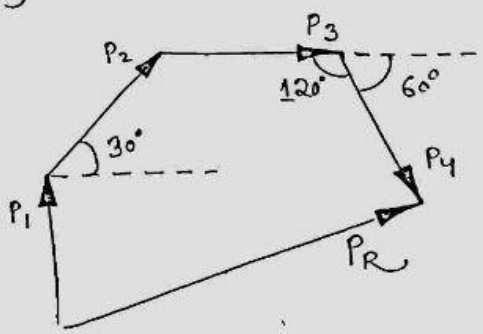
If two forces acting simultaneously at a point are represented in magnitude and direction by two sides of a triangle taken in order, then the third side of the triangle (taken in reverse order) represents the resultant of the two forces in magnitude and direction.

Polygon Law of Forces :⇒

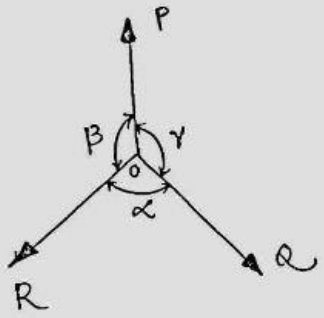
⇒ Polygon Law of forces states that if number of coplanar concurrent forces acting simultaneously on a body be represented in magnitude & direction by the sides of a polygon taken in order, then closing side of the polygon will represent force in magnitude & direction, taken in opposite order.

Let us consider a body on which 4-coplanar forces are acting as shown,





3. **Lami's Theorem** :⇒ It states that, "If a body is in equilibrium under the action of three concurrent and non-collinear forces, then each force is proportional to the sine of the angle between the other two forces.



Mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

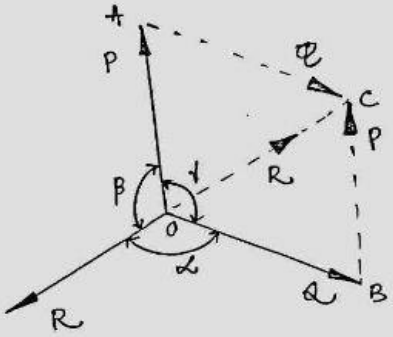
Where, P, Q, R are given forces

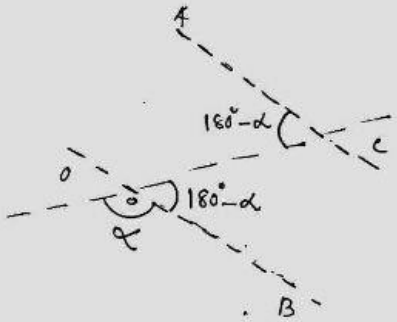
α = angle between Q & R

β = angle between P & R

γ = angle between P & Q

Proof,





BC = P , A = Q

$\angle AOC = 180^\circ - \beta$

$\angle BOC = 180^\circ - \alpha = \angle ACO$

$\angle CAO = 180^\circ - (\angle ACO + \angle AOC)$ } ∵ Sum of Δ = 180°

$\Rightarrow \angle CAO = 180^\circ - [(180^\circ - \alpha) + (180^\circ - \beta)]$

$\Rightarrow \angle CAO = 180^\circ - 180^\circ + \alpha - 180^\circ + \beta$

$\Rightarrow \angle CAO = \alpha + \beta - 180^\circ$ ——— eq° (1)

$\therefore \alpha + \beta + \gamma = 360^\circ$

Substracting 180° from both sides,

$$\alpha + \beta + \gamma - 180^\circ = 360^\circ - 180^\circ$$

$$\Rightarrow (\alpha + \beta - 180^\circ) + \gamma = 180^\circ$$

$$\therefore \Rightarrow \angle COA + \gamma = 180^\circ \quad \left.\begin{array}{l}\\\\\end{array}\right\} \text{From eq}^n (1)$$

$$\Rightarrow \angle COA = 180^\circ - \gamma$$

We know that in Δ AOC,

$$\frac{OA}{\sin \angle ACO} = \frac{AC}{\sin \angle AOC} = \frac{OC}{\sin \angle CAO}$$

$$\Rightarrow \frac{OA}{\sin(180^\circ - \alpha)} = \frac{AC}{\sin(180^\circ - \beta)} = \frac{OC}{\sin(180^\circ - \gamma)}$$

$$\Rightarrow \frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma} \quad \left.\begin{array}{l}\\\\\end{array}\right\} \because \sin(180^\circ - \theta) = \sin\theta$$

4. Resolution of Forces :⇒

Method of Resolution for the Resultant Forces –

1) Resolve all the forces horizontally & Find the algebric sum of all the horizontal components i,e ΣH

2) Resolve all the forces Vertically & Find the algebric sum of all the Vertically components i,e ΣV.

3) The resultant R of the given forces will be given by the equation.

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4) The Resultant force will be inclined at an angle θ with the horizontal, such that.

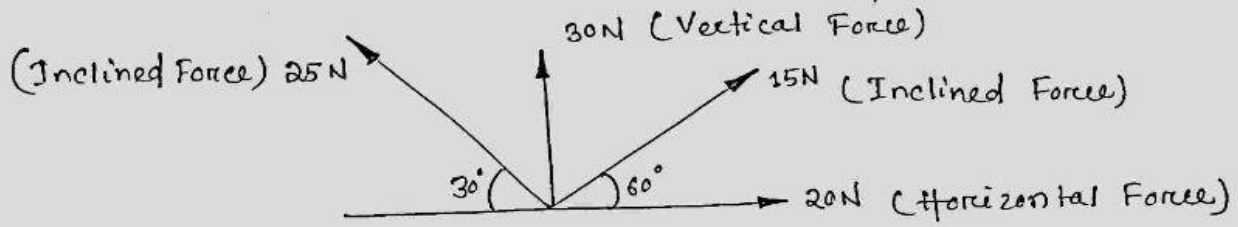
$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

5) The value of the angle θ will vary upon the values of ΣV & ΣH.

6) When ΣV is '+ve' the resultant makes an angle between 0° & 180°. But when ΣV is '-ve', the resultant makes an angle between 180° & 360°.

7) When ΣH is '+ve', the resultant makes an angle between 0° to 90° or 270° to 360°. But when ΣH is '-ve', the resultant makes an angle between 90° to 270°.

Example Find the Resultant for the given force system. Also find the direction & position.



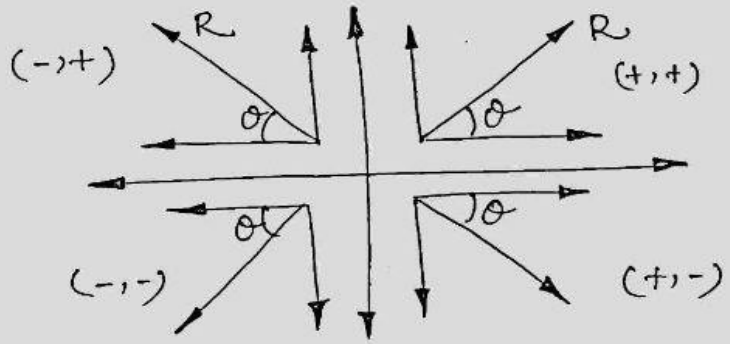
Step-1 : To find the magnitude of Resultant

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

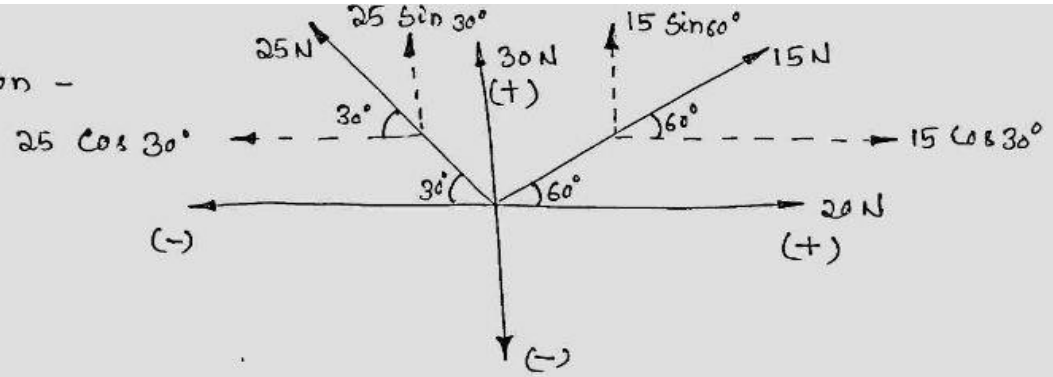
Step-2 : To find direction of the resultant

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

Step 3 : Find the Position of Resultant Force.



Solution -



$\xrightarrow{+}$

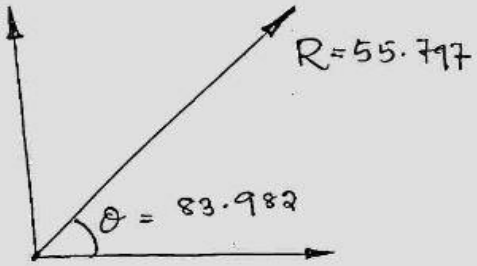
$\Sigma H = 20 + 15\cos 60° - 25\cos 30° = 5.849\ N$ (+ve)

$+\uparrow \Sigma V = 30 + 15\sin 60° + 25\sin 30° = 55.49\ N$ (+ve)

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(5.849)^2 + (55.49)^2}$$

$$= 55.797\ N$$



$$\tan\theta = \left|\frac{\Sigma V}{\Sigma H}\right|$$

$$\Rightarrow \theta = \tan^{-1}\left|\frac{55.49}{5.849}\right|$$

$$\Rightarrow \theta = 83.982°$$

∴ Resultant lies in 1st quadrant.

Equilibrium :⇒

→ Equilibrium is the states of the body when it is subjected to a system of forces. We know that for a system of forces acting on a body the resultant can be determined.

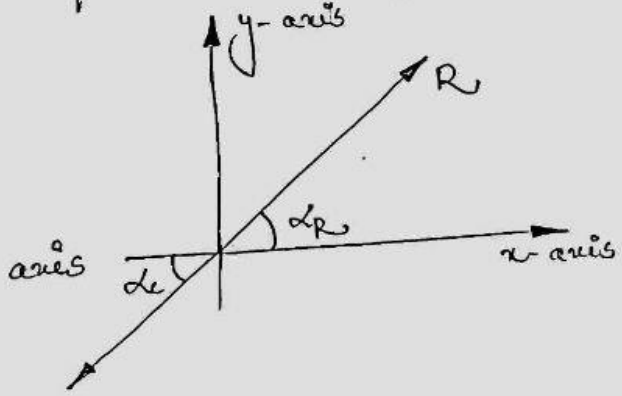
→ By Newton's 2nd Law of Motion the body then should move in the direction of the resultant with some acceleration.

→ If the resultant force is equal to zero it implies that the net effect of the system of forces is zero this represents the state of equilibrium. For a system of coplanar concurrent forces for the resultant to be zero hence,

$$\Sigma H = 0$$
$$\Sigma V = 0$$

Equilibrant :⇒

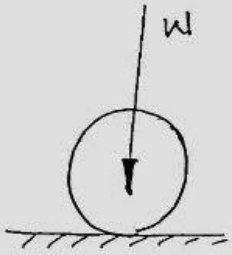
→ Equilibrant is a single force which when added to a system of forces brings the states of equilibrium. Hence this force is of the same magnitude as the resultant but opposite in sense. This is depicted in figure.

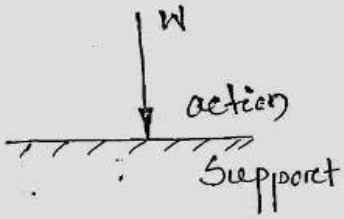


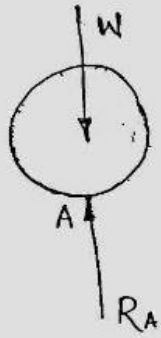
Free Body Diagram

→ Free body diagram is nothing but a sketch which shows the various forces acting on the body. The forces acting on the body could be in form of weight, reactive forces contact forces etc.

Action & Reaction → From the Newton's 3rd Law of motion we know that to every action has always equal and opposite reaction.



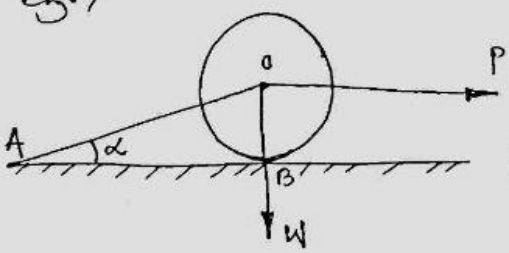


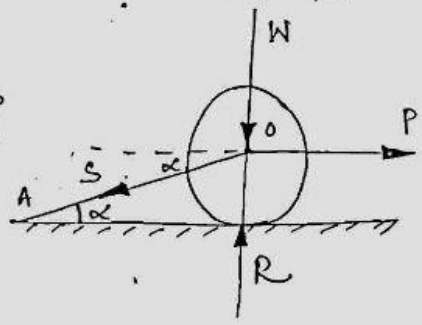


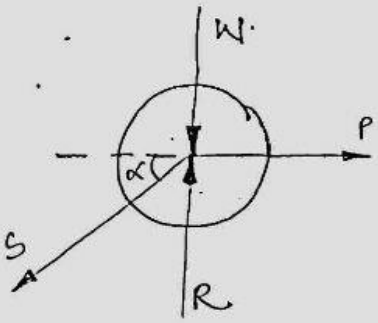
→ The force exerted by the support on the ball is known as reaction.

→ Hence any force on a support causes an equal & opposite from the support. So that action & reaction are two equal & opposite forces.

eg:→







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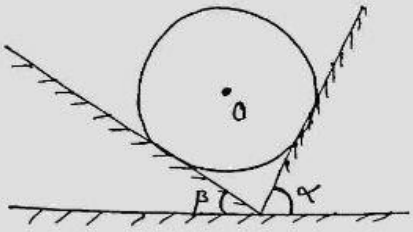

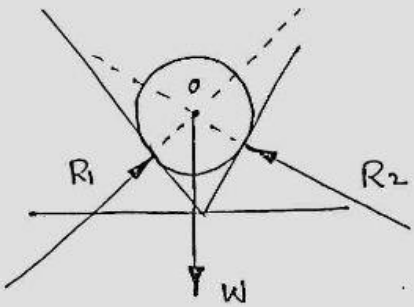

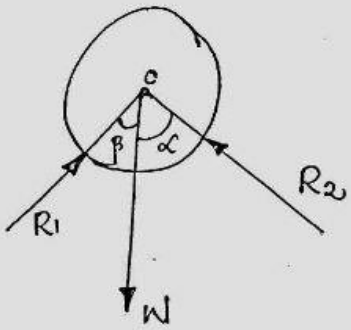

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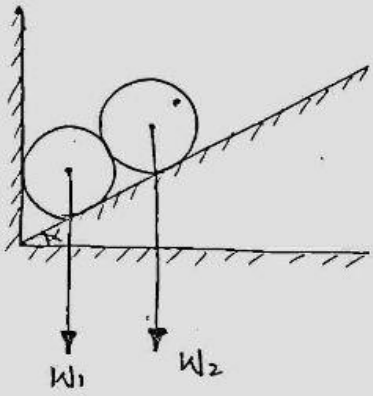
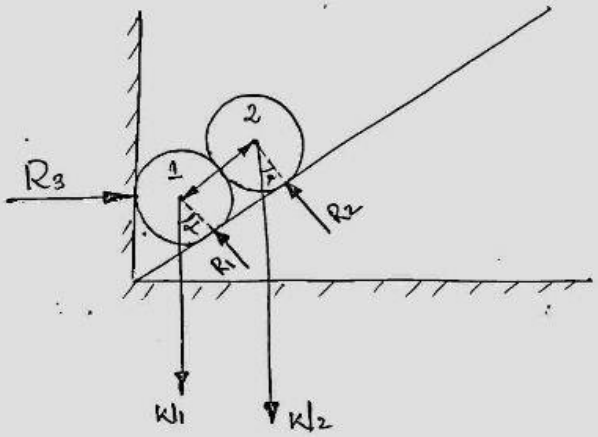
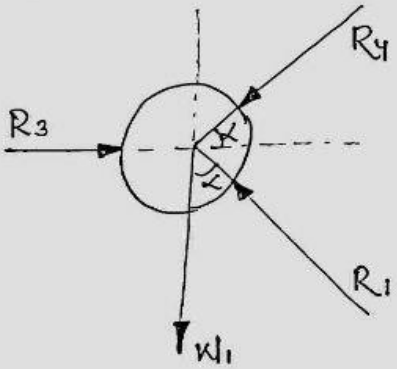
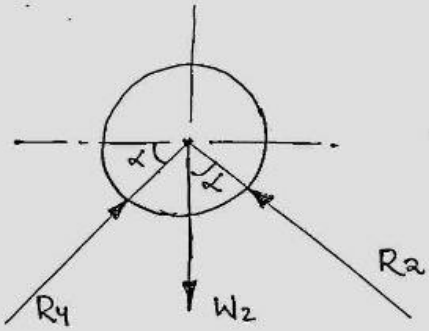




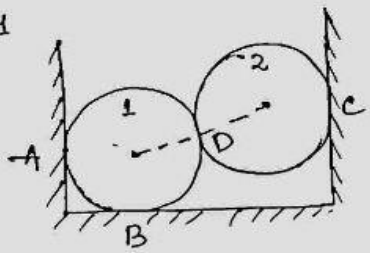
FBD-1

FBD-2

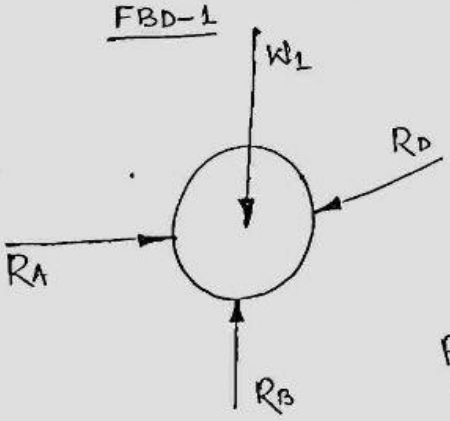




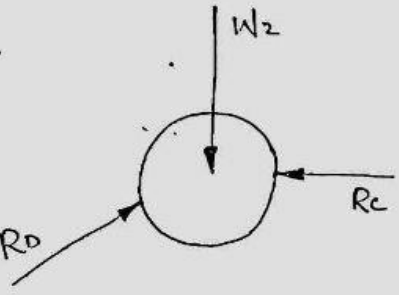
eg-4



FBD-1



FBD-2

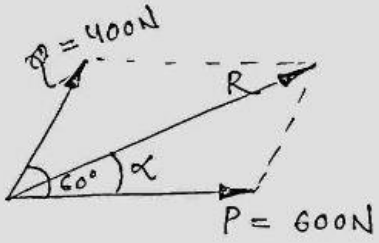


Practice Questions

Q:→ Calculate the resultant of two forces having magnitude 600N & 400N and acting at 60° to one another, when they have (i) Same direction (ii) Opposite direction.

(i) **Same direction →**

Data given, $P = 600N$, $Q = 400N$, $\theta = 60°$



By using Parallelogram law of forces,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{(600)^2 + (400)^2 + 2 \times 600 \times 400 \times \cos 60°}$$

$$= 871.779\,N$$

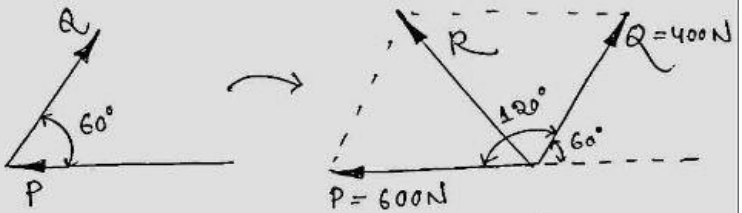
$$\tan\alpha = \left|\frac{Q\sin\theta}{P + Q\cos\theta}\right|$$

$$\Rightarrow \alpha = \tan^{-1}\left|\frac{400 \cdot \sin 60°}{600 + 400\cos 60°}\right|$$

$$\Rightarrow \alpha = \tan^{-1}\left|\frac{346.41}{800}\right|$$

$$\Rightarrow \alpha = 23.413°$$

(ii) **Opposite direction →**



Data given,

$P = 600N$
$Q = 400N$
$\theta = 120°$

{By applying Principle of Transmissibility

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

$$\Rightarrow R = \sqrt{600^2+400^2+2\times600\times400\times\cos120^\circ}$$

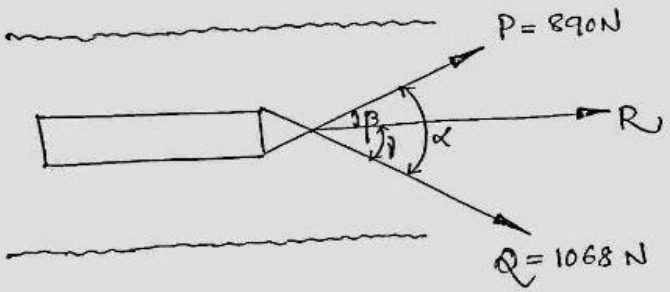
$$\Rightarrow R = 529.15\ N$$

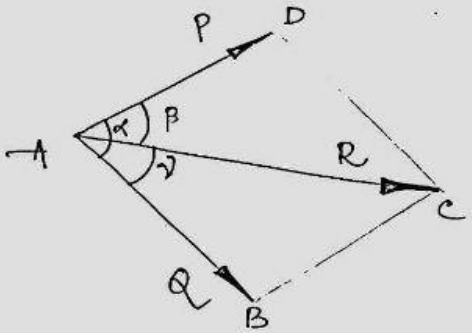
$$\alpha = \tan^{-1}\left|\frac{Q\sin\theta}{P+Q\cos\theta}\right|$$

$$\Rightarrow \alpha = \tan^{-1}\left|\frac{400\sin120^\circ}{600+400\cos120^\circ}\right|$$

$$\Rightarrow \alpha = 40.89^\circ$$

Q. A boat is moved uniformly along a canal by two forces $P = 890N$ & $Q = 1068\ N$ acting under an angle $\alpha = 60^\circ$ as shown in figure. Determine the magnitude of the resultant pull on the boat & the angles β & γ as shown in the figure.





Data given, $P = 890\ N$, $Q = 1068\ N$, $\alpha = 60^\circ$

To find $R = ?$, $\beta = ?$, $\gamma = ?$

By applying parallelogram law,

$$R = \sqrt{P^2+Q^2+2PQ\cos\alpha}$$

$$= \sqrt{890^2+1068^2+2\times890\times1068\times\cos60^\circ}$$

$$= 1698.01\ N$$

$$\tan\gamma = \frac{P\sin\alpha}{Q + P\cos\alpha}$$

$$\Rightarrow \tan\gamma = \frac{890\sin 60^\circ}{1068 + 890\cos 60^\circ}$$

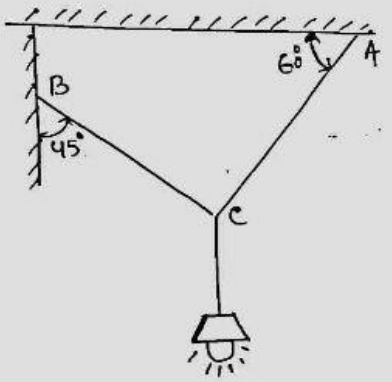
$$\Rightarrow \gamma = \tan^{-1}(0.5094)$$

$$\Rightarrow \gamma = 27^\circ$$

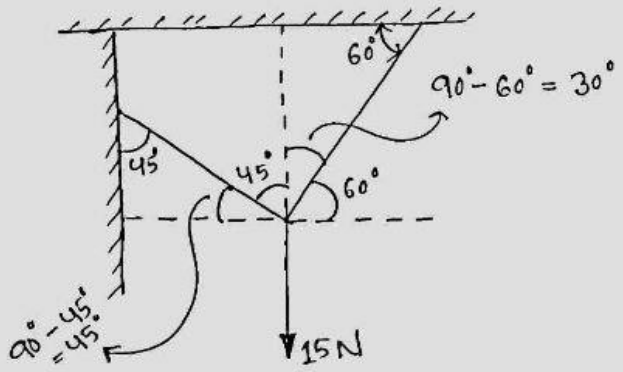
$$\alpha = \beta + \gamma$$

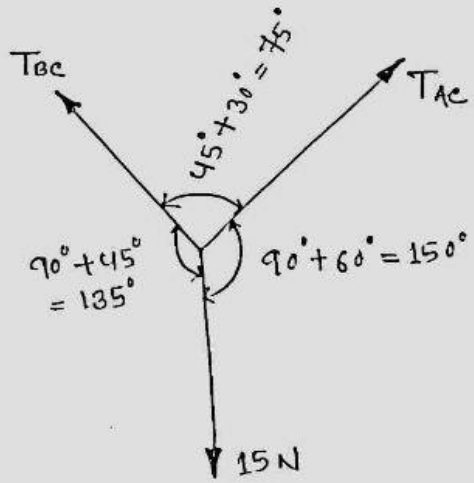
$$\Rightarrow \beta = \alpha - \gamma = 60^\circ - 27^\circ = 33^\circ$$

Q:- An electric light fixture weighting 15 N hangs from a point 'c' by two strings AC & BC. The string AC is inclined at 60° to the horizontal & BC at 45° to the horizontal as shown in figure. By using Lami's theorem Determine the forces in the string AC & BC.



Sol:-





Data given, $W = 15\,N$

T_{AC} = Force in the string AC

T_{BC} = Force in the string BC

By applying Lami's theorem at point 'C',

$$\frac{15}{\sin 75^\circ} = \frac{T_{AC}}{\sin 135^\circ} = \frac{T_{BC}}{\sin 150^\circ}$$

$$\Rightarrow \frac{15}{\sin 75^\circ} = \frac{T_{AC}}{\sin(180^\circ - 45^\circ)} = \frac{T_{BC}}{\sin(180^\circ - 30^\circ)}$$

$$\Rightarrow \frac{15}{\sin 75^\circ} = \frac{T_{AC}}{\sin 45^\circ} = \frac{T_{BC}}{\sin 30^\circ}$$

$$\therefore \frac{15}{\sin 75^\circ} = \frac{T_{AC}}{\sin 45^\circ}$$

$$\Rightarrow T_{AC} = \frac{15 \times \sin 45^\circ}{\sin 75^\circ}$$

$$\Rightarrow T_{AC} = 10.98\,N$$

$$\therefore \frac{15}{\sin 75^\circ} = \frac{T_{BC}}{\sin 30^\circ}$$

$$\Rightarrow T_{BC} = \frac{15 \times \sin 30^\circ}{\sin 75^\circ}$$

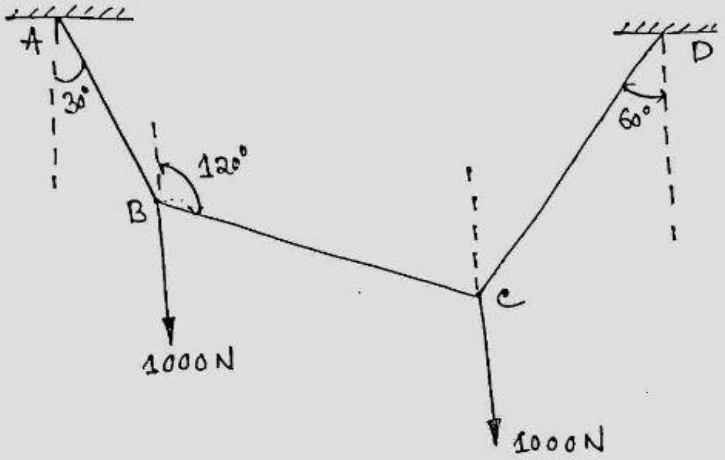
$$\Rightarrow T_{BC} = 7.76\,N$$

$\therefore$ $T_{AC} = 10.98\,N$

$T_{BC} = 7.76\,N$

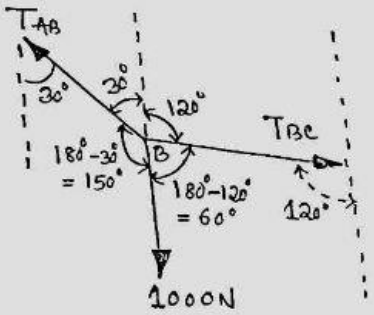
Q: A string ABCD, attached to fixed points A & D has two equal weights of 1000 N attached to it at B & C. The weight rest with the portion

AB & CD inclined at an angle as shown in figure. Find the tensions in the portions AB, BC, CD of the string, if the inclination of the portion BC with the vertical is 120°.

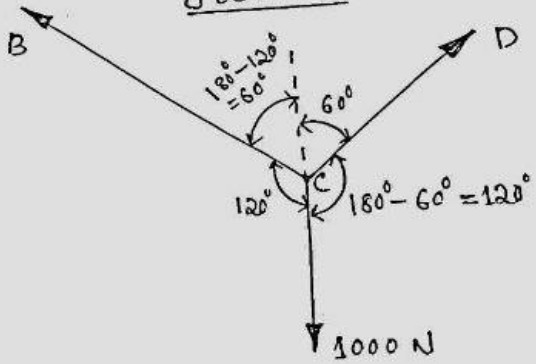


Solution :→

Joint B



Joint - C



Data given, $W_B = 1000N$

$W_C = 1000\ N$

To find, T_{AB} = Tension in the portion AB

T_{BC} = Tension in the portion BC

T_{CD} = Tension in the portion CD

By applying Lami's equation at joint B,

$$\frac{T_{AB}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{1000}{\sin 150^\circ}$$

$$\Rightarrow \frac{T_{AB}}{\sin 60^\circ} = \frac{T_{BC}}{\sin(180^\circ - 30^\circ)} = \frac{1000}{\sin(180^\circ - 30^\circ)}$$

$$\therefore \frac{T_{AB}}{\sin 60^\circ} = \frac{1000}{\sin 30^\circ}$$

$$\Rightarrow T_{AB} = \frac{\sin 60^\circ \times 1000}{\sin 30^\circ}$$

$$\Rightarrow T_{AB} = 1732\ N$$

$$\therefore \frac{T_{BC}}{\sin 30^\circ} = \frac{1000}{\sin 30^\circ}$$

$$\Rightarrow T_{BC} = \frac{1000 \times \sin 30^\circ}{\sin 30^\circ}$$

$$\Rightarrow T_{BC} = 1000\ N$$

Again applying Lami's equation at joint 'C',

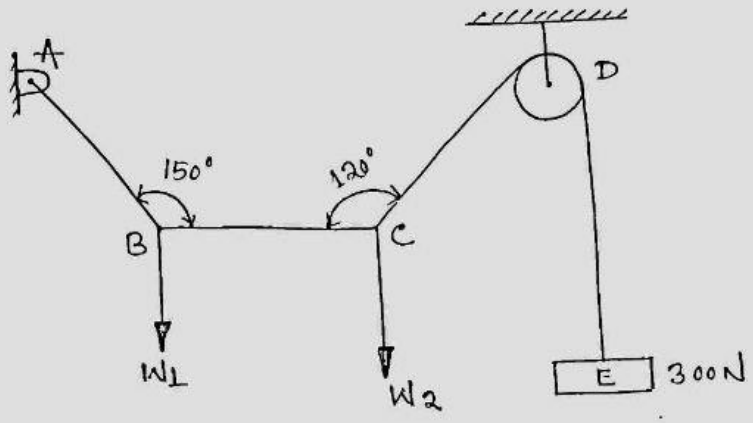
$$\frac{T_{BC}}{\sin 120^\circ} = \frac{T_{CD}}{\sin 120^\circ} = \frac{1000}{\sin 120^\circ}$$

$$\therefore \frac{T_{CD}}{\sin 120^\circ} = \frac{1000}{\sin 120^\circ}$$

$$\Rightarrow T_{CD} = \frac{1000 \times \sin 120^\circ}{\sin 120^\circ}$$

$$\Rightarrow T_{CD} = 1000\ N$$

Q:- A light string ABCDE, whose extremity A is fixed has weights W_1 & W_2 attached to it at B & C. It passes round a small smooth peg at D carrying a weight of 300 N at the free end E as shown in figure.

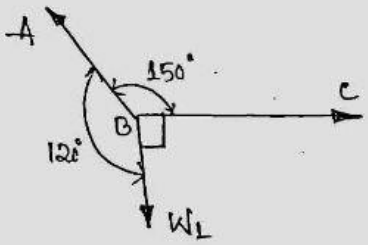


If in the equilibrium position, BC is horizontal & AB & CD make 150° & 120° with BC.

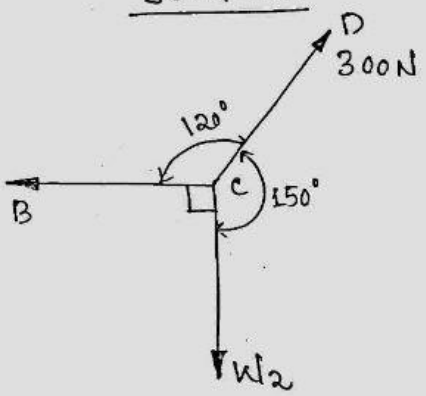
Find (i) Tension in the portion AB, BC & CD of the string and

(ii) Magnitude of W_1 & W_2.

Joint - B



Joint - C



Data given, $W_E = 300N$

$T_{CD} = T_{DE} = 300N$

Let us split up the string ABCD into 2 parts.

To find, Tensions in the portion AB, BC, CD

$T_{CD} = T_{DE} = 300N$

$T_{AB} = ?$

$T_{BC} = ?$

Applying Lami's Theorem at 'C',

$$\frac{T_{BC}}{\sin 150^\circ} = \frac{W_2}{\sin 120^\circ} = \frac{300}{\sin 90^\circ}$$

$$\Rightarrow \frac{T_{BC}}{\sin(180^\circ - 30^\circ)} = \frac{W_2}{\sin(180^\circ - 60^\circ)} = \frac{300}{1}$$

$$\Rightarrow \frac{T_{BC}}{\sin 30^\circ} = \frac{W_2}{\sin 60^\circ} = 300$$

$$\Rightarrow \frac{T_{BC}}{\sin 30^\circ} = 300$$

$$\Rightarrow T_{BC} = 300 \times \sin 30^\circ$$

$$\Rightarrow T_{BC} = 150\ N \quad (\text{Tension})$$

$$\therefore \frac{W_2}{\sin 60^\circ} = 300$$

$$\Rightarrow W_2 = 300 \sin 60^\circ$$

$$\Rightarrow W_2 = 259.8\ N$$

Again applying Lami's equation at 'B'

$$\frac{T_{AB}}{\sin 90^\circ} = \frac{W_1}{\sin 150^\circ} = \frac{T_{BC}}{\sin 120^\circ}$$

$$\Rightarrow \frac{T_{AB}}{1} = \frac{W_1}{\sin(180^\circ - 30^\circ)} = \frac{150}{\sin(180^\circ - 60^\circ)}$$

$$\Rightarrow \frac{T_{AB}}{1} = \frac{150}{\sin 60^\circ}$$

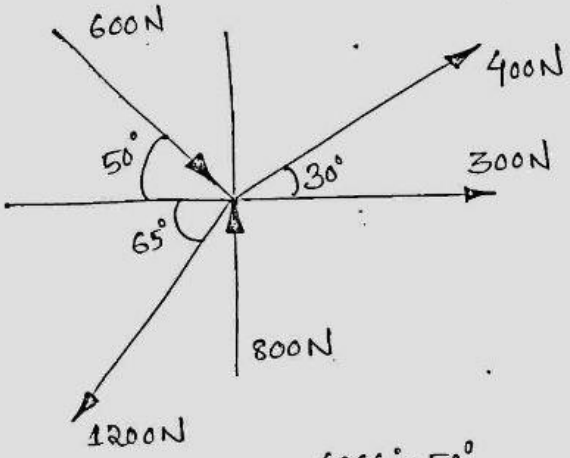
$$\Rightarrow T_{AB} = 173.2\ N \quad (\text{Tension})$$

$$\frac{W_1}{\sin 30^\circ} = \frac{150}{\sin 60^\circ}$$

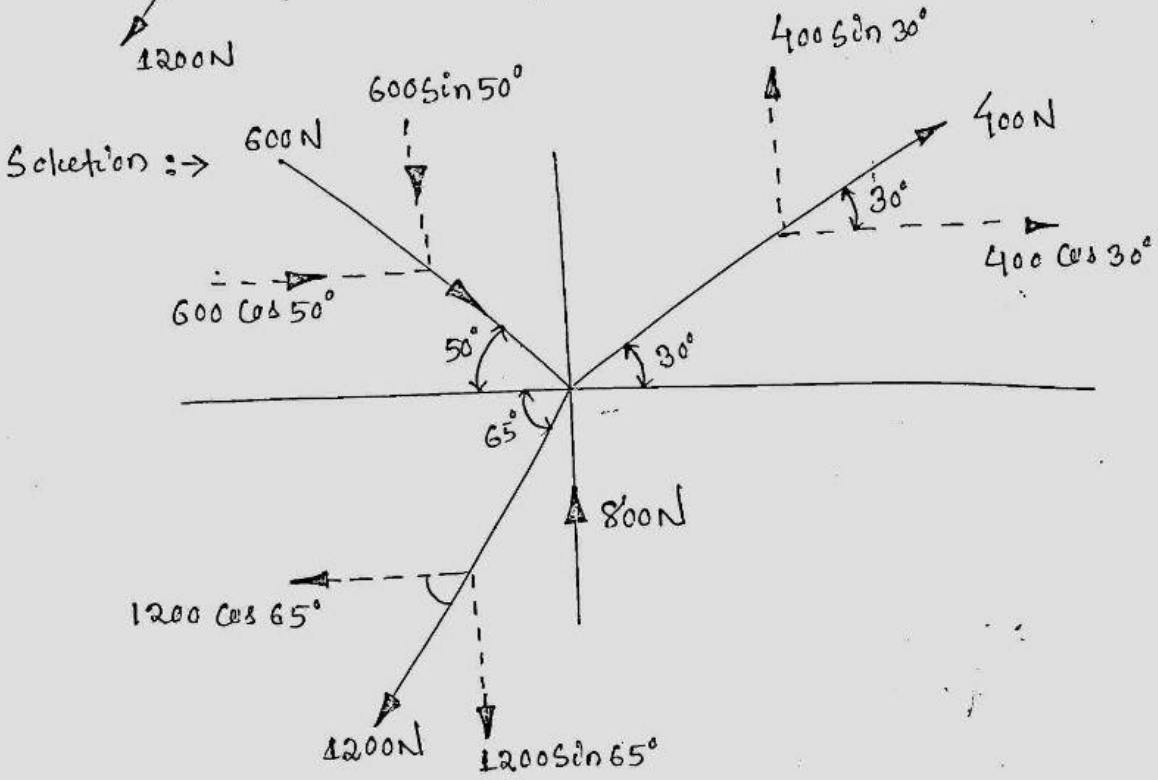
$$\Rightarrow W_1 = \frac{150 \times \sin 30^\circ}{\sin 60^\circ}$$

$$\Rightarrow W_1 = 86.6\ N$$

Q:



Solution :->



$$\xrightarrow[\Sigma H]{+} = 300 + 400 \cos 30^\circ + 600 \cos 50^\circ - 1200 \cos 65^\circ$$

$$= 524.94 \text{ N}$$

$$+\uparrow \Sigma V = 800 - 1200 \sin 65 - 600 \sin 50^\circ + 400 \sin 30^\circ$$

$$= -547.196 \text{ N}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

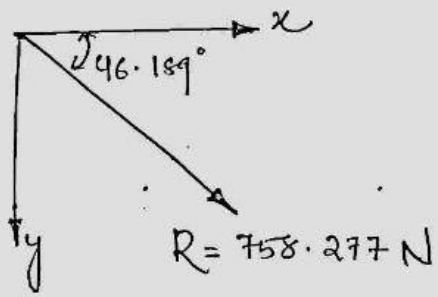
$$\Rightarrow R = \sqrt{(524.94)^2 + (-547.196)^2}$$

$$\Rightarrow R = 758.277 \text{ N}$$

$$\tan\theta = \left|\frac{\Sigma V}{\Sigma H}\right|$$

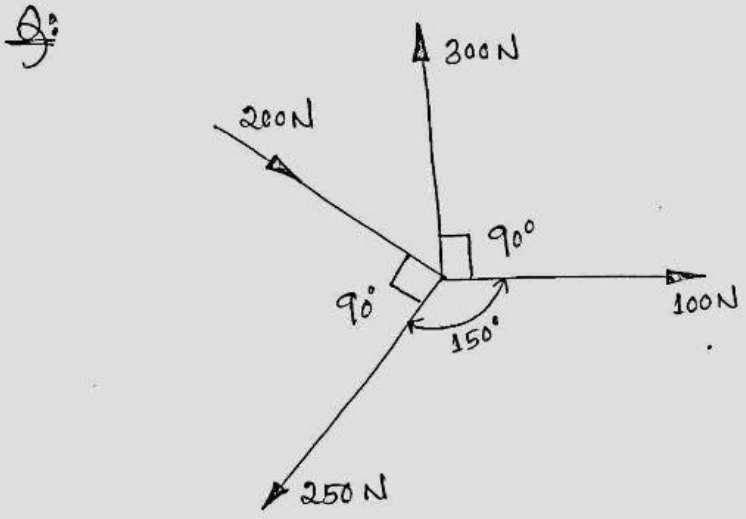
$$\Rightarrow \theta = \tan^{-1}\left|\frac{547.196}{527.94}\right|$$

$$\Rightarrow \theta = 46.189^\circ$$

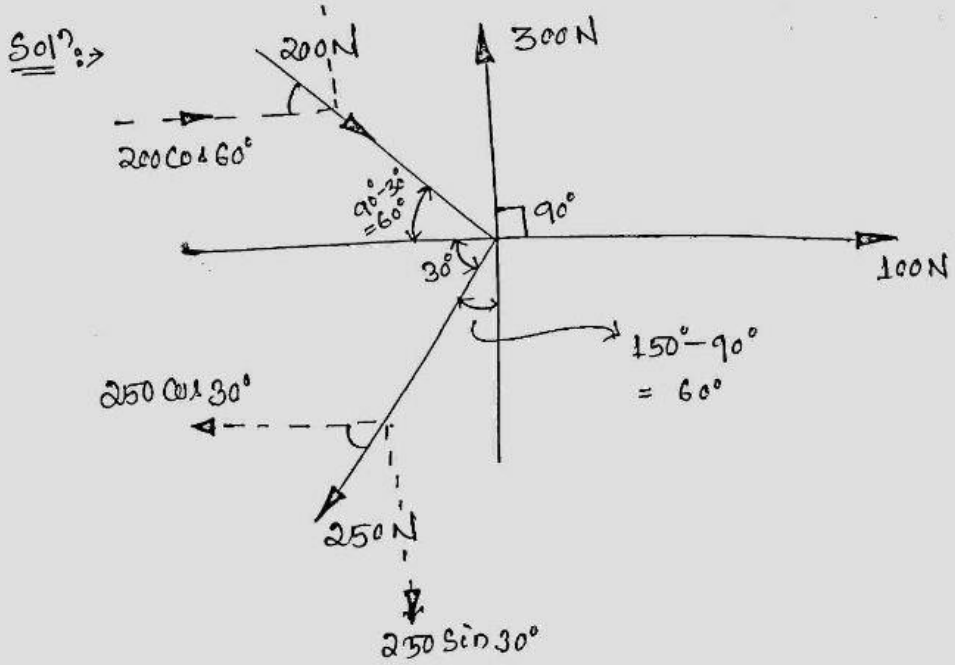


Since ΣH is +ve, ΣV is -ve, So Resultant is lies in 4th quadrant.

Q:



Sol?:>



$$\xrightarrow{+}\Sigma H = 100 + 200\cos 60^\circ - 250\cos 30^\circ$$

$$= -16.506\,N$$

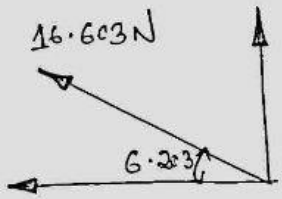
$$+\uparrow \Sigma V = 300 - 200\sin 60^\circ - 250\sin 30^\circ$$

$$= 1.794\,N$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$\Rightarrow R = \sqrt{(-16.506)^2 + (1.794)^2}$$

$$\Rightarrow R = 16.603\,N$$



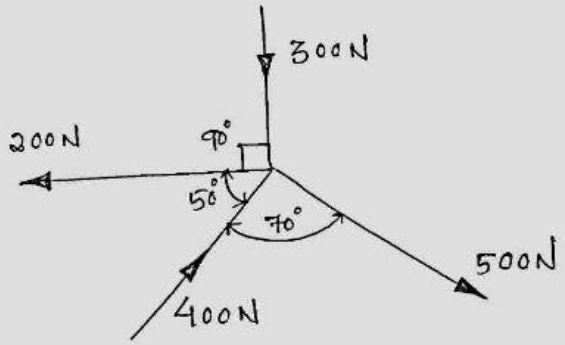
$$\theta = \tan^{-1}\left|\frac{\Sigma V}{\Sigma H}\right|$$

$$\Rightarrow \theta = \tan^{-1}\left|\frac{1.794}{16.506}\right|$$

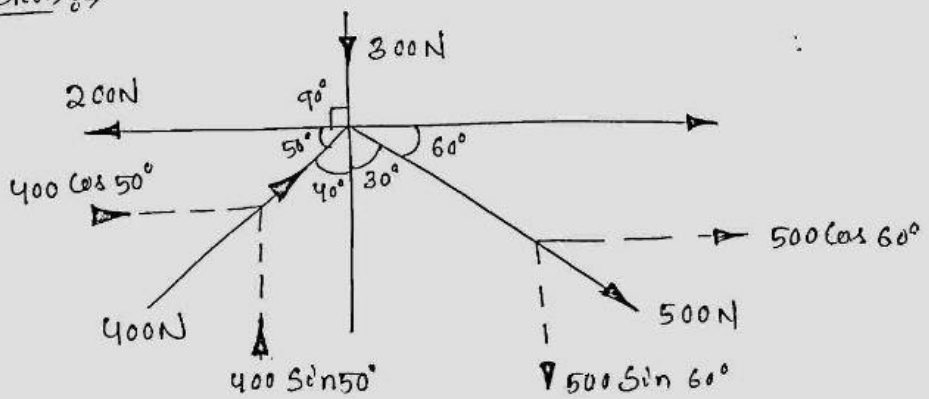
$$\Rightarrow \theta = 6.203^\circ$$

∴ Since ΣH is "-ve" & ΣV is "+ve". Hence the Resultant force lies in 2nd quadrant

Q:



Solution :→



$$\overset{+}{\rightarrow}\Sigma H = 400\cos 50^\circ + 500\cos 60^\circ - 200$$

$$= 307.115\ N$$

$$+\uparrow \Sigma V = 400\sin 50^\circ - 500\sin 60^\circ - 300$$

$$= -426.59\ N$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(307.115)^2 + (-426.59)^2}$$

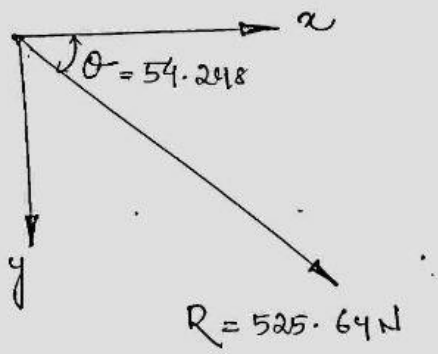
$$= \sqrt{9431.62 + 181979.02}$$

$$= 525.64\ N$$

$$\theta = \tan^{-1}\left|\frac{\Sigma V}{\Sigma H}\right|$$

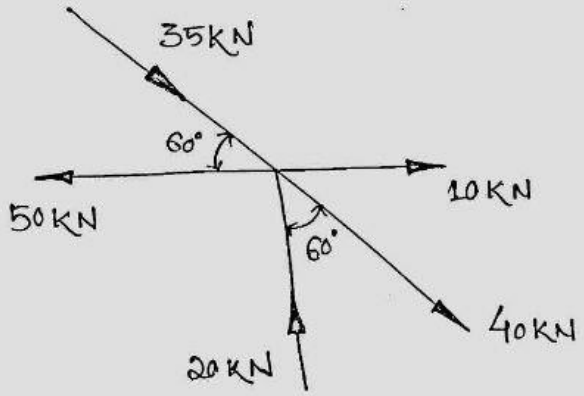
$$= \tan^{-1}\left|\frac{-426.59}{307.115}\right|$$

$$= 54.248$$

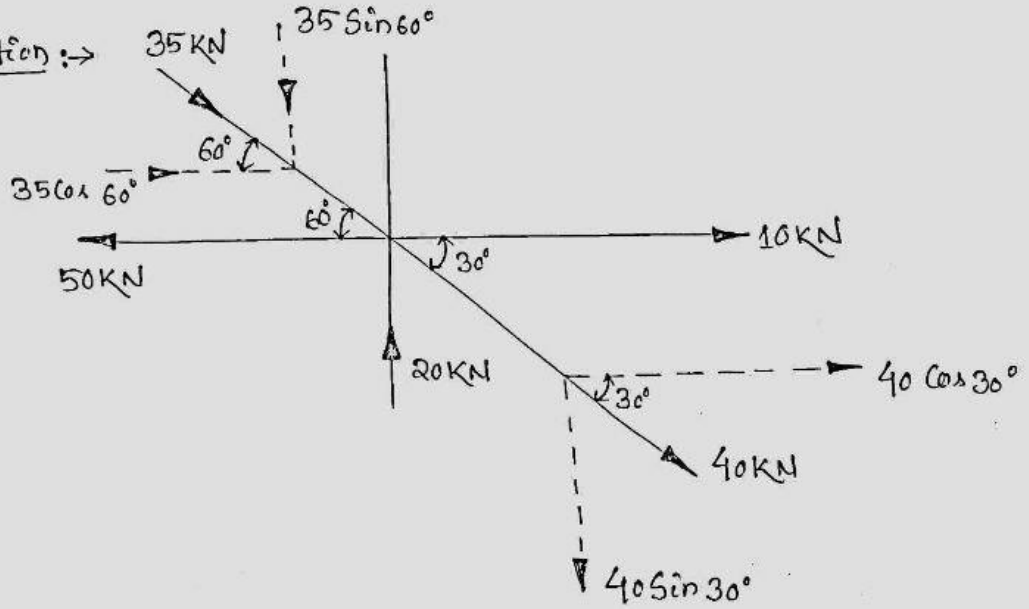


Since ΣH is +ve & ΣV is -ve. So Resultant lies in 4th quadrant.

Q:



Solution :→



$$\xrightarrow{+} \Sigma H = 10 + 40\cos 30^\circ + 35\cos 60^\circ - 50$$

$$= 12.141 \text{ KN}$$

$$+\uparrow \Sigma V = 20 - 40\sin 30^\circ - 35\sin 60^\circ$$

$$= -30.310 \text{ KN}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$\Rightarrow R = \sqrt{(12.141)^2 + (-30.310)^2}$$

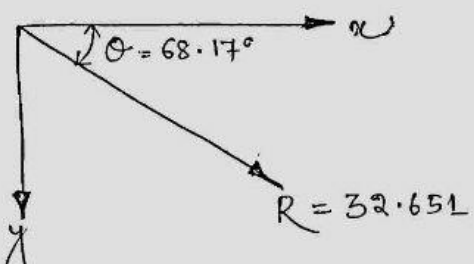
$$\Rightarrow R = 32.651 \text{ KN}$$

$$\theta = \tan^{-1}\left|\frac{\Sigma V}{\Sigma H}\right|$$

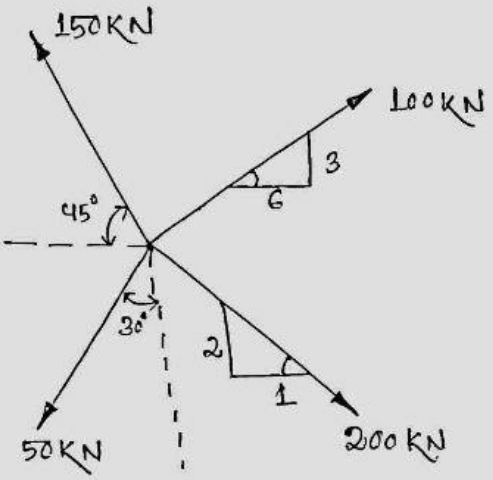
$$\Rightarrow \theta = \tan^{-1}\left|\frac{-30.310}{12.141}\right|$$

$$\Rightarrow \theta = 68.17^\circ$$

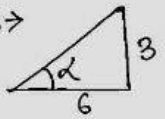
Since ΣH is +ve & ΣV is -ve. The resultant lies in 4th quadrant.



Q:



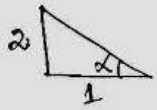
Solution:→



$\tan\alpha = \frac{P}{b} = \frac{3}{6}$

$\Rightarrow \alpha = \tan^{-1}(3/6)$

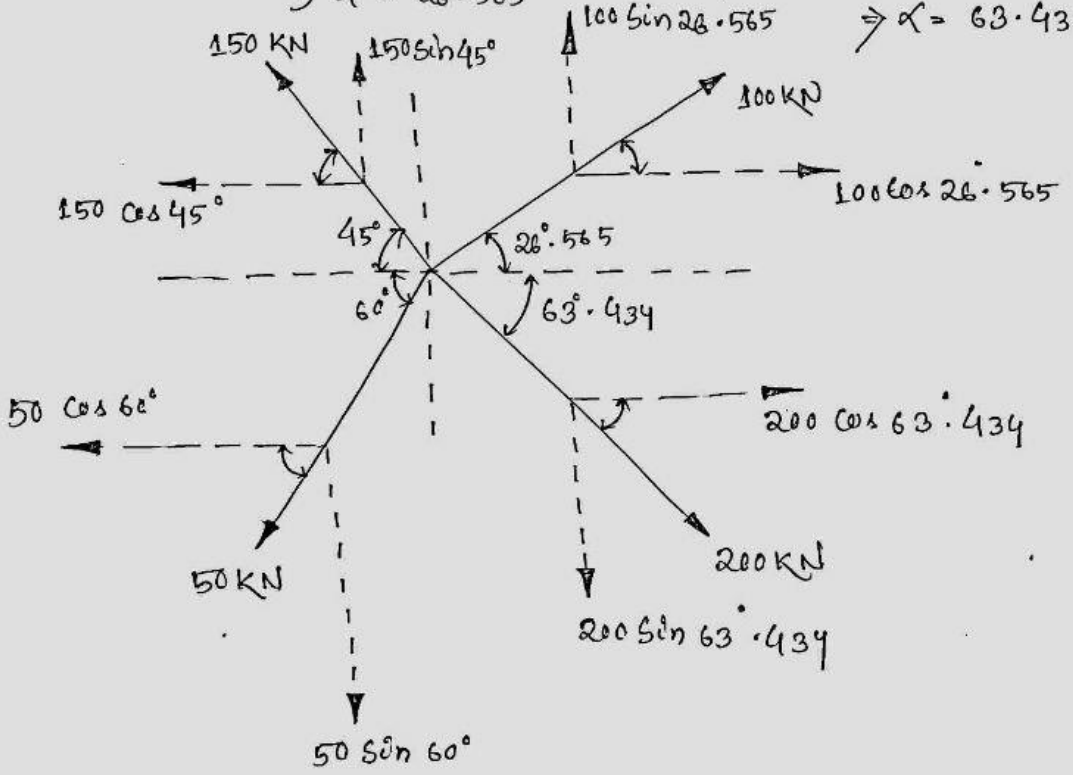
$\Rightarrow \alpha = 26.565°$



$\tan\alpha = \frac{2}{1}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{2}{1}\right)$

$\Rightarrow \alpha = 63.434°$



$\xrightarrow{+} \Sigma H = 100 \cos 26.565 + 200 \cos 63.434 - 50 \sin 60° - 150 \cos 45°$

$= 47.822 \text{ KN}$

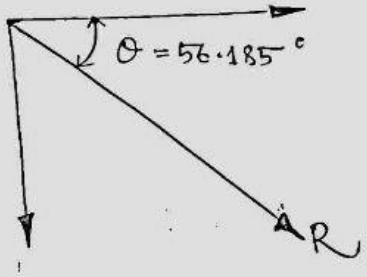
$+\uparrow \Sigma V = 100 \sin 26.565 + 150 \sin 45^\circ - 50 \cos 30^\circ - 200 \sin 63.434$

$= 18.02 KN$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(47.822)^2 + (71.397)^2}$

$= 85.932 KN$



$\theta = \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right|$

$\Rightarrow \theta = \tan^{-1} \left| \frac{71.397}{47.822} \right|$

$\Rightarrow \theta = 56.185^\circ$

Since ΣH is +ve & ΣV is -ve. Resultant lies in 4th quadrant

Q: The following forces act at a point,

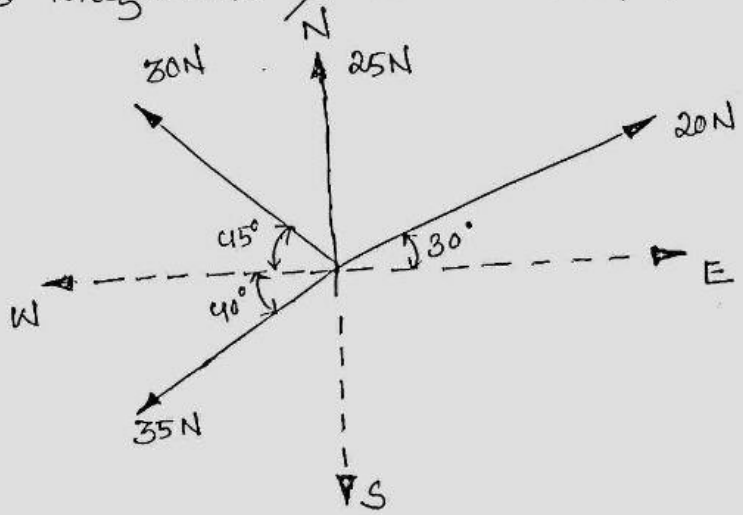
(i) 20N inclined at 30° towards North of East

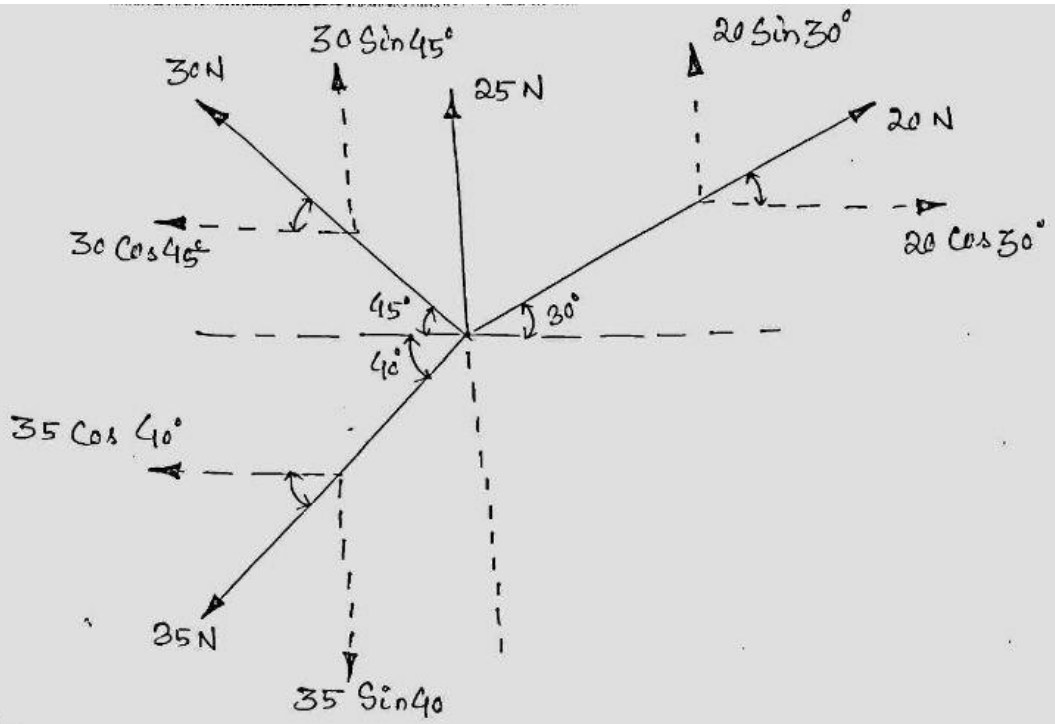
(ii) 25 N towards North

(iii) 30N towards North - West

(iv) 35 N inclined at 40° towards South of West

Find the magnitude & direction of the resultant force.





$\xrightarrow{+}\ \Sigma H = 20\cos 30^\circ - 30\cos 45^\circ - 35\cos 40^\circ$

$= -30.70\ N$

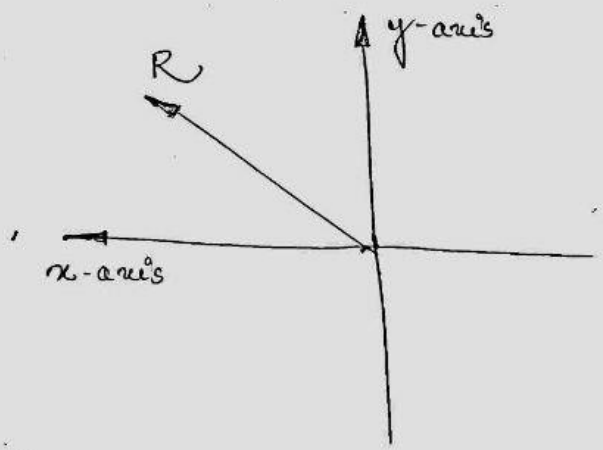
$+\uparrow \Sigma V = 25 + 30\sin 45^\circ - 35\sin 40^\circ + 20\sin 30^\circ$

$= 33.71\ N$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$\Rightarrow R = \sqrt{(-30.70)^2 + (33.71)^2}$

$\Rightarrow R = 45.6\ N$



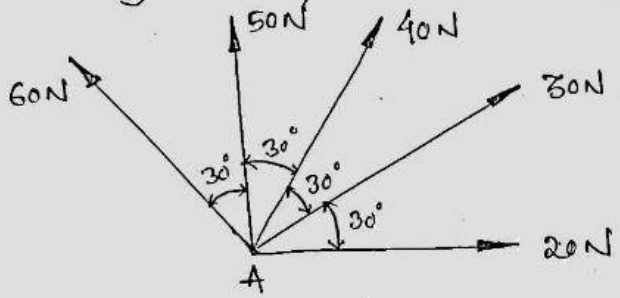
$\tan\theta = \left|\frac{\Sigma V}{\Sigma H}\right|$

$\Rightarrow \theta = \tan^{-1}\left|\frac{33.7}{-30.7}\right|$

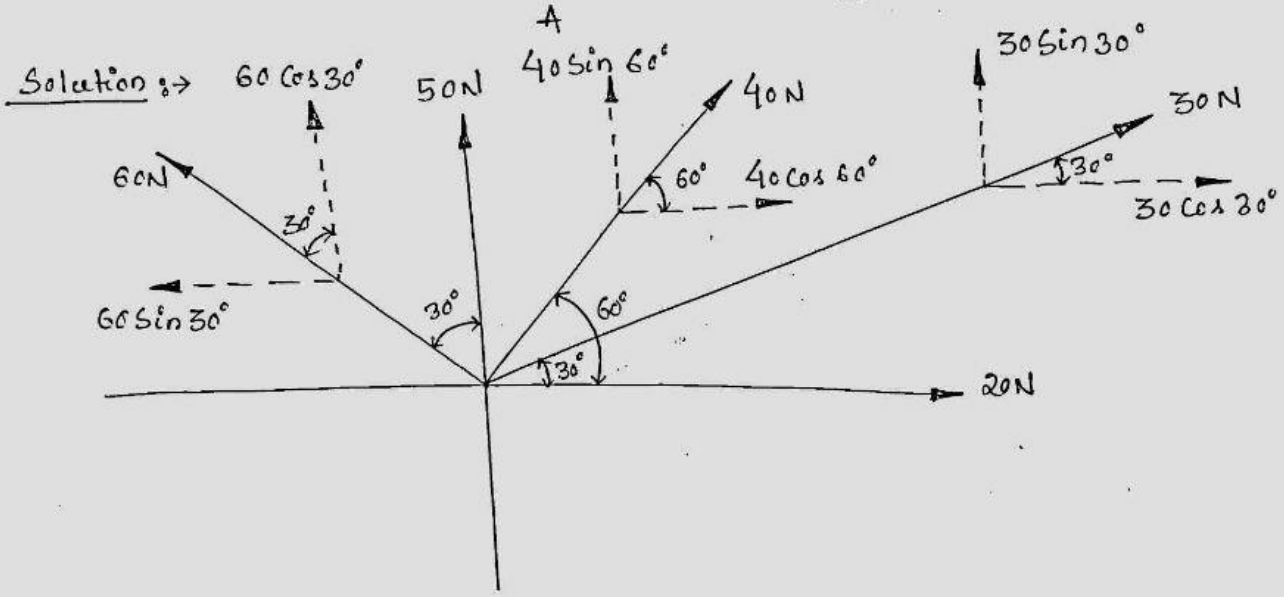
$\Rightarrow \theta = -1.098$ or 47.7°

$\therefore$ Resultant lies in 2nd quadrant

Q: The forces 20N, 30N, 40N, 50N, 60N are acting at one of the angular points of a regular hexagon towards the other five angular points taken in order. Find the magnitude & direction of the resultant force.



Solution :→



$$\xrightarrow{+} \Sigma H = 20 + 30\cos 30^\circ + 40\cos 60^\circ - 60\sin 30^\circ = 36\,N$$

$$+\uparrow \Sigma V = 50 + 60\cos 30^\circ + 40\sin 60^\circ + 30\sin 30^\circ = 151.6\,N$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

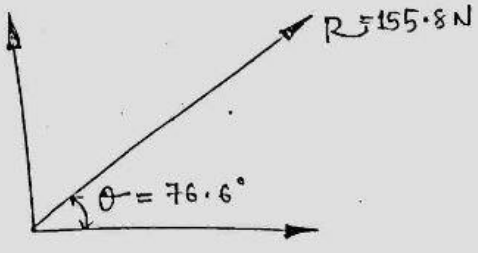
$$= \sqrt{(36.0)^2 + (151.6)^2}$$

$$= 155.8\,N$$

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{151.6}{36}$$

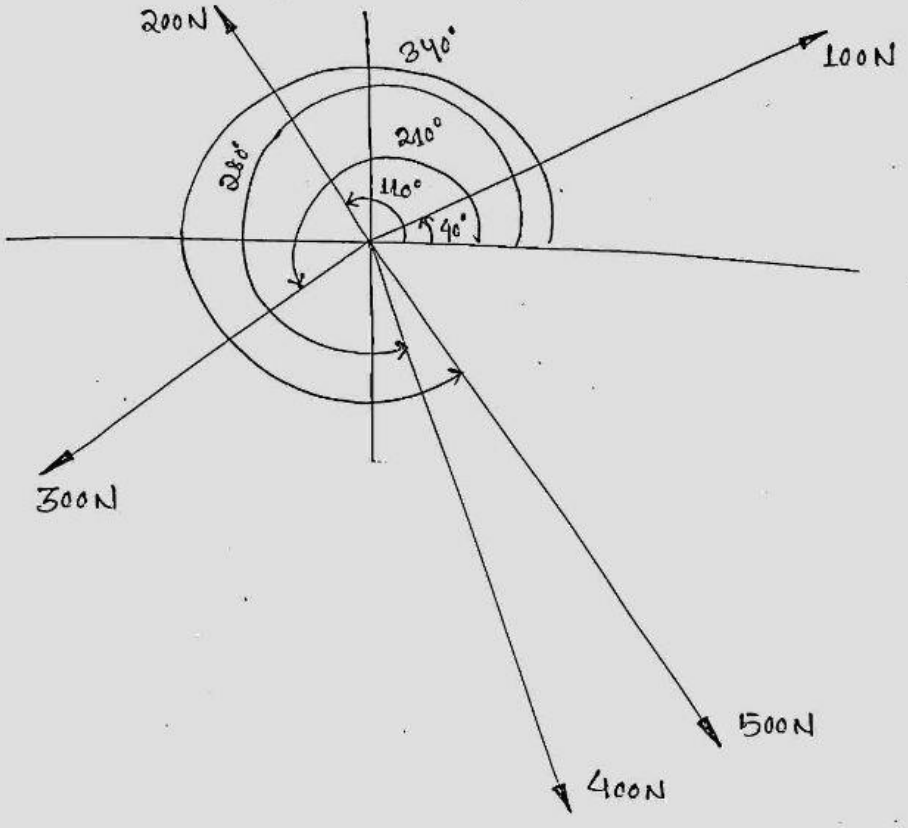
$$\Rightarrow \theta = \tan^{-1}\left(\frac{151.6}{36}\right)$$

$$\Rightarrow \theta = 76.6^\circ$$



Since both the value of ΣH & ΣV are positive, therefore the Resultant Force lies between 0° to 90° or 1st quadrant.

Q: Five force 100N, 200N, 300N, 400N & 500N are acting at an angle of 40°, 110°, 220°, 280° & 340° in the anti-clockwise direction from x-axis at a point. All acting away from the point. Find the resultant force in magnitude direction.

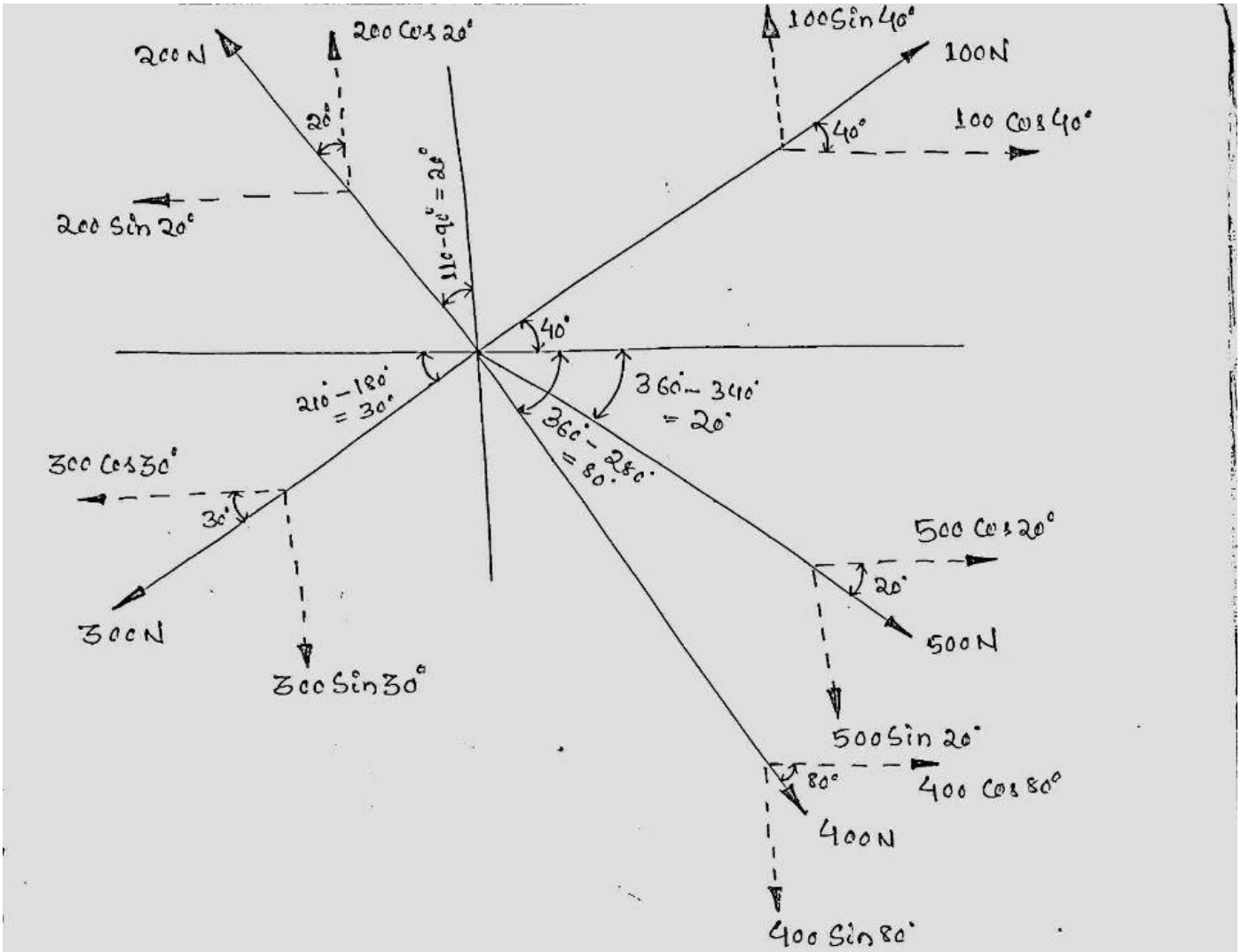


$\xrightarrow{+} \Sigma H = 100 \cos 40° + 500 \cos 20° + 400 \cos 80° - 200 \sin 20° - 300 \cos 30°$

$= 217.69\ N$

$+\uparrow \Sigma V = 100 \sin 40° + 200 \cos 20° - 300 \sin 30° - 400 \sin 80° - 500 \sin 20°$

$= -462.71\ N$



$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

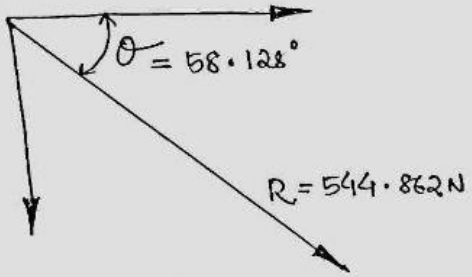
$$= \sqrt{(287.69)^2 + (-462.71)^2}$$

$$= 544.862 \text{ N}$$

$$\tan\theta = \left|\frac{\Sigma V}{\Sigma H}\right|$$

$$\Rightarrow \theta = \tan^{-1}\left|\frac{-462.71}{287.69}\right|$$

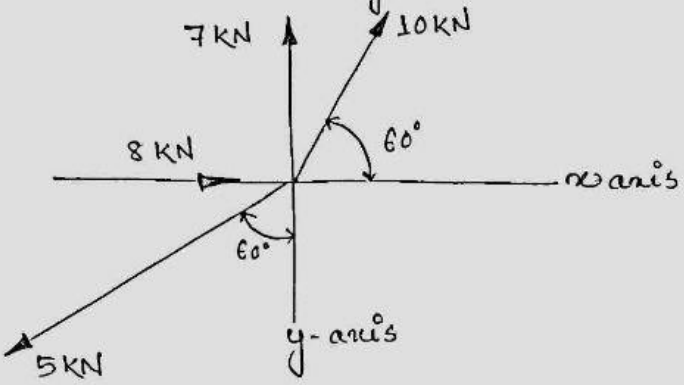
$$\Rightarrow \theta = 58.128$$



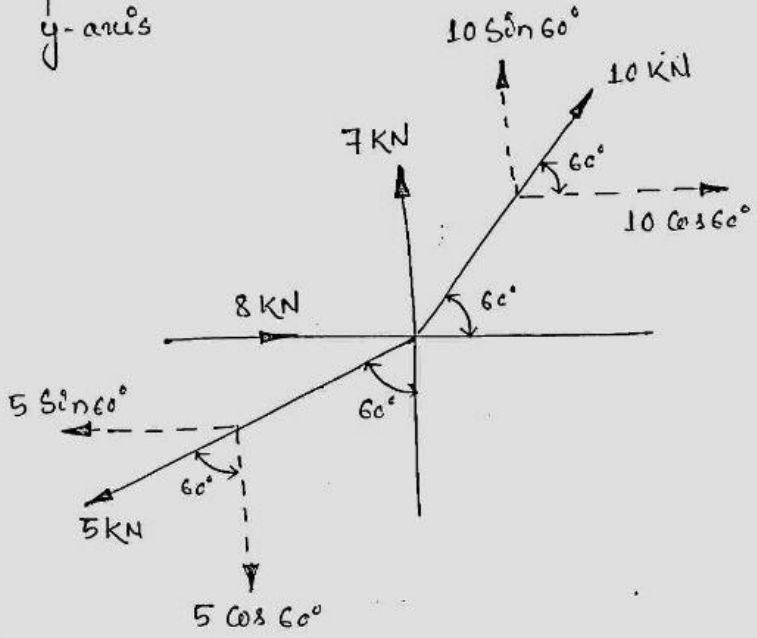
∴ Since ΣH is +ve and ΣV is -ve. So the Resultant lies in 4th quadrant.

Class Test

Q:> Find resultant of a force system shown in Figure.



Solution :-



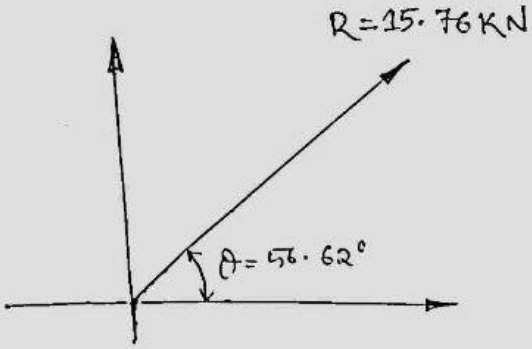
$$\xrightarrow{+} \Sigma H = 10\cos 60^\circ - 5\sin 60^\circ + 8 = 8.66\,KN$$

$$+\uparrow \Sigma V = 7 - 5\cos 60^\circ + 10\sin 60^\circ = 13.16\,KN$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(8.66)^2 + (13.16)^2}$$

$$= 15.76\,KN$$

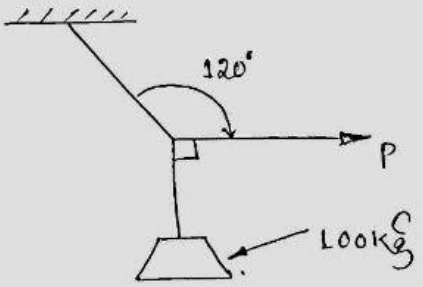


$$\tan\theta = \left|\frac{\Sigma V}{\Sigma H}\right|$$

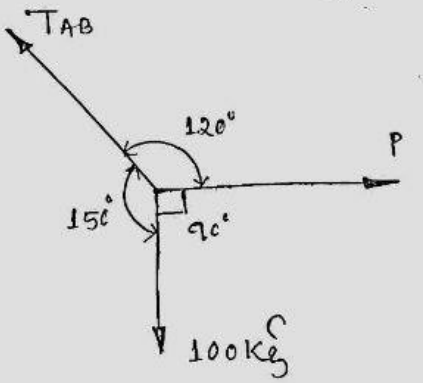
$$\Rightarrow \theta = \tan^{-1}\left|\frac{13.16}{8.66}\right|$$

$$\Rightarrow \theta = 56.62^\circ$$

Q:→ Find the magnitude of the force P, required to keep the 100 Kg mass in the position by string as shown in the figure.



Ans



By applying Lami's theorem,

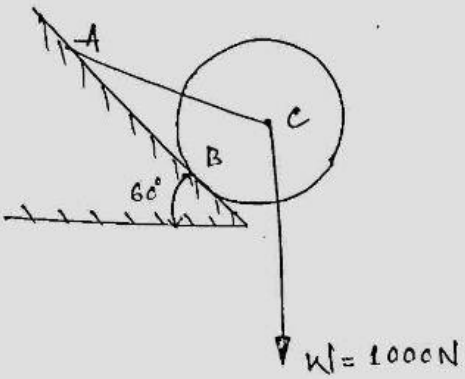
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

$$\Rightarrow \frac{P}{\sin 150^\circ} = \frac{T_{AB}}{\sin 90^\circ} = \frac{100}{\sin 120^\circ}$$

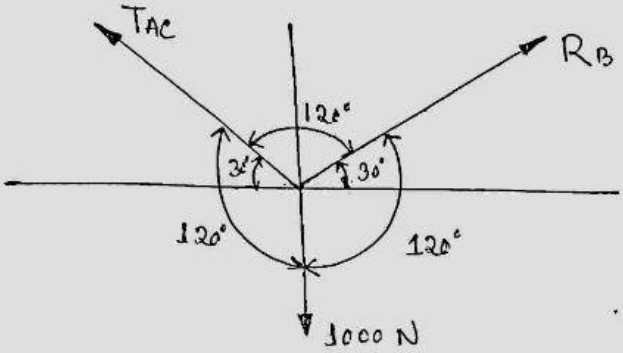
$$\therefore P = 566.38\ N$$

$$T_{AB} = 1132.76\ N$$

Q:→ A cylindrical roller 600mm diameter and weight 1000N is resting on a smooth inclined surface, tied firmly by a rope AC of length 600mm as shown in figure. Find tension in rope & reaction at B.



ans FBD



By applying Lami's theorem.

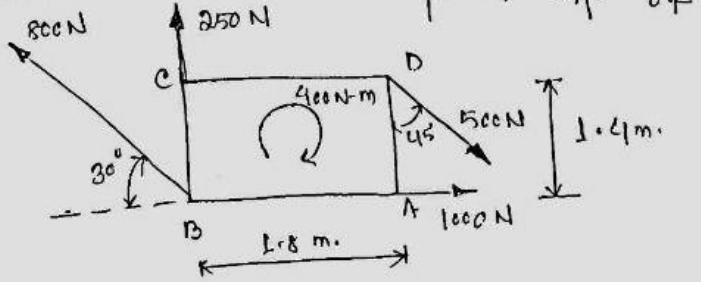
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

$$\therefore \frac{T_{AC}}{\sin 120^\circ} = \frac{R_B}{\sin 120^\circ} = \frac{1000}{\sin 120}$$

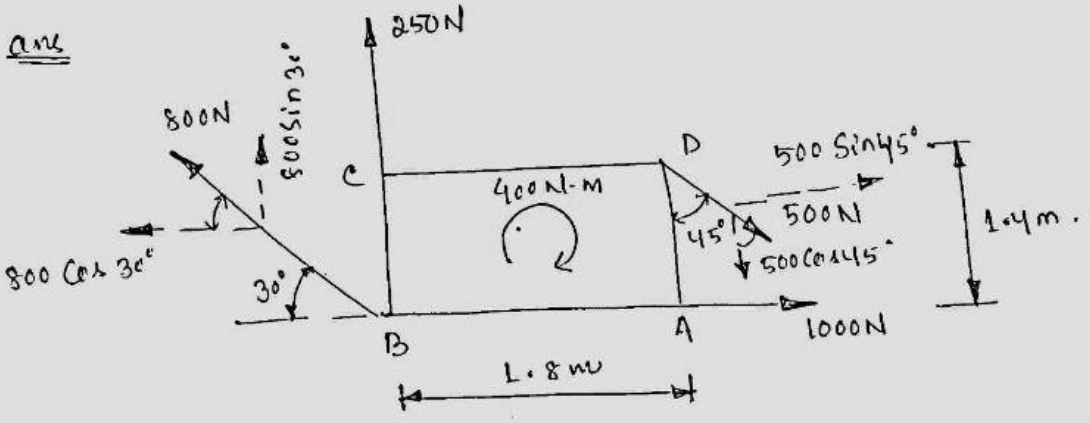
$$\therefore T_{AC} = 1000\,N$$

$$R_B = 1000\,N$$

Q: For a coplanar, non-concurrent force system shown in figure. determine magnitude, direction, position with reference to point A of resultant force.



ans



$$\xrightarrow{+} \quad \Sigma H = 500 \sin 45^\circ - 800 \cos 30^\circ + 1000$$

$$= 660.73 \text{ N}$$

$$+\uparrow \Sigma V = -500 \cos 45^\circ + 850 + 800 \sin 30^\circ$$

$$= 896.45 \text{ N}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(660.73)^2 + (896.45)^2}$$

$$= 1113.64 \text{ N}$$

$$\tan\theta = \left| \frac{896.45}{660.73} \right|$$

$$\Rightarrow \theta = \tan^{-1} \left| \frac{896.45}{660.73} \right|$$

$$\Rightarrow \theta = 53.61^\circ$$