FOUNDATION ENGINEERING [RCI6D001] LECTURE NOTES B. TECH 3RD YEAR – VI SEM (2024-25)



DEPARTMENT OF CIVIL ENGINEERING

MODERN ENGINEERING

&

MANAGEMENT STUDIES

UNIT – I

SOIL EXPLORATION

Introduction

A fairly accurate assessment of the characteristics and engineering properties of the soils at a site is essential for proper design and successful construction of any structure at the site.

The field and laboratory investigations required to obtain the necessary data for the soils for this purpose are collectively called as the Soil Exploration.

The choice of foundation and its depth, the bearing capacity, settlement analysis and such other important aspects depend very much upon the various engineering properties of the foundation soils involved.

Soil exploration may be needed not only for the design and construction of new structures, but also for deciding upon the remedial measures if an existing structure shows signs of distress after construction.

The design and construction of highways and airport pavements will also depend upon the characteristics of the soil strata upon which they are to be aligned.

The primary objectives of soil exploration are as follows

1). Determination of the nature of the deposits of soil.

2). Determination of the depth and thickness of the various soil strata and their extent in the horizontal direction.

3). The location of ground water and fluctuations in Ground Water Table.

4). Obtaining soil and rock samples from the various strata.

5). Determination of the engineering properties of the soil and rock strata that affect the performance of the structure, and

6). Determination of the in-situ properties by performing the various field tests.

Soil exploration involves broadly the following:

1). Planning of a program for soil exploration.

2). Collection of disturbed and undisturbed soil and rock samples from the drilled holes in the field. The number and depths of holes depend upon the in-situ conditions of soil and the nature of project.

3). Conducting all the necessary in-situ tests for obtaining the strength and compressibility characteristics of the soil and either directly or indirectly.

4). Study of ground water conditions and collection of water samples for chemical analysis.

5). Conducting all the necessary tests on the samples of soil/rock and water collected.

6). Conducting the Geophysical explorations, if required.

7). Preparation of drawings, charts, etc.

8). Analysis of data collected.

9). Preparation of report.

The in-situ field tests for a project may consist of any one or more of the following tests:

- 1. Standard penetration test in boreholes (SPT).
- 2. Static cone penetration tests (CPT).
- 3. Vane shear test (VST).
- 4. Plate load test (PLT).
- 5. Permeability test (PT).

Site investigation

'Site Investigation' refers to the procedure of determining the surface and sub-surface conditions in the area of proposed construction.

It is essential for assessing the suitability of the site for the proposed engineering work and for preparing adequate and economical design.

Site investigation is also necessary for selecting the construction materials and for deciding construction methods to be adopted.

Thus the term 'Site Investigation' has a broader connotation than 'Soil Exploration' and includes the later.

Site investigation may be needed not only for new works, but also for deciding the remedial measures to be taken if the existing work shows signs of distress after construction.

In general the purpose of site investigation is to obtain the information about the surface conditions as well as sub-surface conditions existing at the site.

Accordingly the site investigation also includes sub-soil exploration which constitutes a major component of site investigation.

Usually the cost of site investigation and sub-soil exploration will be less than 1% of the total cost of the entire project.

Site investigation in one form or the other is generally required for every big engineering project. Information about the surface and sub-surface features is essential for the design of structures and for planning the construction techniques.

Site investigations consist of determining the profile of the natural soil deposit at the site, taking the soil samples and determining the engineering properties of the soils. It also includes in-situ testing of the soils.

Site investigations are generally done to obtain the information that is useful for one (or) more of the following purposes:

1). To select the type and depth of foundation for a given structure.

2). To determine the bearing capacity of the soil.

3). To estimate the probable maximum and differential settlements.

- 4). To establish the ground water level and to determine the properties of water.
- 5). To predict the lateral earth pressure against retaining walls and abutments.
- 6). To select the suitable construction techniques.
- 7). To predict and to solve potential foundation problems.
- 8). To ascertain the suitability of the soil as a construction material.
- 9). To investigate the safety of the existing structures and to suggest the remedial measures.

During the site investigation, the relative information is obtained by drilling holes, taking the soil samples and determining the index and engineering properties of the soil. In-situ tests are also conducted to determine the properties of the soils in natural conditions.

Preliminary steps of site investigation

Site investigation may involve the following preliminary steps:

1). Reconnaissance 2). Study of maps and 3). Aerial photography

1. Reconnaissance

Reconnaissance involves an inspection of the site and study of the topographical features. This will yield useful information about the soil and ground water conditions and also help in deciding the plan and programme of sub-soil or sub-surface exploration.

During reconnaissance the features may be studied: local topography, excavations, cuttings, quarries, escarpments, landslides or erosions, fills, water levels in wells and streams, flood marks and drainage pattern, etc. Aerial reconnaissance is also undertaken if the area is large and the project is major one.

Reconnaissance gives a preliminary idea of the soil and other conditions involved at the site and its value should not be underestimated.

However, further study may be avoided if reconnaissance reveals the unsuitability of the site for the proposed work due to any glaring reasons.

2. Study of maps

Information about surface and sub-surface conditions in an area is frequently available in the form of maps. Such sources in India are the Survey of India and Geological Survey of India, which provide topographical maps, often called 'Topographical sheets'.

The geological study is also essential. The main purpose of this study is to determine the nature of the deposits underlying the site.

The types of soil and rock likely to be encountered can be determined and the method of exploration most suited to the site may be selected. Faults, folds, cracks, fissures, dikes, sills and caves and such other defects in rock and soil strata may be indicated.

Seismic potential or potential seismic activity is a major factor in structural design in many regions of the world, especially in the construction of major structures like dams and nuclear power plants.

Maps are available showing the earth quake zones of different degrees of vulnerability.

3. Aerial Photography

Aerial photography is now a fairly well developed method by which site investigation may be conducted for any major project.

Photographs are obtained in sequence by flying in more or less straight lines across the site with two-thirds overlap in the direction of flight and one-quarter overlap between the successive flight lines.

Air photo interpretation consists of estimating underground conditions by relating landform development and plant growth to geology as reflected in aerial photographs and also identification of all the natural and man-made features.

The features include topography, stream patterns, erosion details, vegetation, man-made features and micro details in topography such as sink-holes, rock outcrops and accumulation of boulders. Each of these features is used to associate with a particular type of rock or soil stratum.

Air photo interpretation requires a thorough knowledge of geology, geomorphology, agriculture and hydrology. The technique, though highly specialized, is a valuable preview and supplement to site reconnaissance.

Planning a sub-surface exploration programme

The planning of a sub-surface or sub-soil exploration programme depends on the type and importance of the structure to be built and upon the variability of the soil strata at the proposed site.

The important items which need to be determined by a sub-soil exploration programme are : the depth, thickness, extent and composition of each of the strata, the depth of the rock and the depth of the ground water table.

The planning should include a site plan of the area, a layout plan of proposed structures with column locations and expected loads and location of bore holes and other field tests.

A carefully planned programme of boring and sampling is the cost import point to be considered in any sub-soil exploration work.

The extent of sub-surface exploration is closely related to the relative cost of the investigations and that of the entire project for which it is undertaken,

In general, the more detailed the investigations are done, the more is known about the sub-surface conditions.

As a result, the greater economy can be achieved in the construction of the project because the element of uncertainty is considerably reduced.

The extent of investigation would also depend upon the location of the project. A small house in an already built-up area would not require much exploration.

On the other hand, if the house is to be built in a newly developed area, a detailed investigation would be required to ascertain the location of different soil strata and their physical characteristics.

If a multi-storeyed building is to be constructed, extensive sub-surface exploration would be necessary. These buildings impose very heavy loads and the zone of influence is also very deep.

It would, therefore be more desirable to invest more amount on sub-surface exploration than to over design the building and to make it costlier.

Planning of sub-surface exploration programme is a difficult task. Besides a thorough knowledge of soil engineering, it requires experience and engineering judgment. Resourceful and intelligent personnel trained in the principles of geology and geotechnical engineering are necessary.

Sometimes, the exploration programme has to be changed as the investigation progress. As the variability of soil strata is found to increase, the extent of investigation is also increased.

On the other hand, if the site is found to be underlain by uniform deposits, the extent of investigation is decreased.

In general, the aim of sub-surface exploration investigation should be to get the maximum information that is useful in the design and construction of the project at a minimum cost.

The cost of sub-surface exploration generally varies between 0.05 to 0.2% of the total cost of the entire structure. In some unusual conditions, the cost may be even up to 1%.

Stages in sub-surface explorations

The sub-surface explorations are generally carried out in three stages:

1. Reconnaissance 2. Preliminary explorations and 3. Detailed explorations

(1). Reconnaissance.

Site reconnaissance is the first step in a sub-surface or sub-soil exploration programme. It includes a visit to the site and to study the maps and other relevant records.

It helps in deciding future programme of site investigations, scope of work, methods of exploration to be adopted, types of samples to be taken and laboratory testing and in-situ testing.

(2). Preliminary explorations.

The aim of preliminary explorations is to determine the depth, thickness, extent and composition of each soil stratum at the site. The depth of bed rock and the ground water table is also determined.

The preliminary explorations are generally in the form of a few borings or test pits. Tests are conducted with cone penetrometers and sounding rods to obtain information about the strength and compressibility of soils.

Geophysical methods are also used in preliminary explorations for locating the boundaries of different soil strata.

(3). Detailed explorations.

The purpose of detailed explorations is to determine the engineering properties of the soils in different strata. It includes an extensive boring programme, soil sampling and testing of soil samples in the laboratory.

Field tests, such as vane shear tests, plate load tests and permeability tests are conducted to determine the properties of soils in natural state.

The tests for determination of dynamic properties are also carried out, if required.

For complex projects involving heavy structures, such as bridges, dams, multi-storey buildings, it is essential to have detailed explorations.

However, for small projects, especially at sites where the strata are uniform, detailed investigations may not be required.

The design of such projects is generally based on the data collected during reconnaissance and preliminary explorations.

Sub-surface exploration - Reconnaissance

The geotechnical engineer makes a visit to the site for a careful visual inspection in reconnaissance. The information about the following features is obtained in reconnaissance. 1). The general topography of the site, the existence of drainage ditches and dumps of debris and sanitary fills.

2). Existence of settlement cracks in the structures already built near the site.

3). The evidence of landslides, creep of slopes and the shrinkage cracks.

4). The stratification of soils as observed from deep cuts near the site.

5). The location of high flood marks on the nearby buildings and bridges.

6). The depth of ground water table as observed in the wells.

7). Existence of springs, swamps, etc. at the site.

8). The drainage pattern existing at the site.

9). Types of vegetation existing at the site. The type of vegetation gives a clue about the nature of soil at the site.

10). Existence of underground water mains, power conduits, etc. at the site.

In addition to making site visits, the geotechnical engineer should study geological maps, aerial photographs, topographical sheets and soil maps.

The maps give a lot of information about the geological character of the area.

The geotechnical engineer should also get information about the type of structure to be built and its proposed use.

In the case of multi-storeyed building, the information about the column loads and their approximate locations should be obtained.

In the case of bridges, the span length and the load carried by the piers and abutments should be ascertained. In the case of a dam, the geotechnical engineer should get information about the type of dam its length, base width and other salient characteristics.

The information obtained during reconnaissance is helpful in evolving a suitable sub-surface investigation programme.

Depth of exploration

The depth of exploration required at a particular site depends upon the degree of variation of the subsurface data in the horizontal and vertical directions.

It is not possible to fix the number, disposition and depth of borings without making a few preliminary borings (or) soundings at the site.

The depth of exploration is governed by the depth of the influence zone.

The depth of influence zone depends upon the type of the structure, intensity of loading, shape and disposition of the loaded area, the soil profile and the physical characteristics of the soil.

The depth upto which the stress increment due to superimposed loads can produce significant settlement and shear stress is known as the "**significant depth**".

The depth of exploration should be at least equal to the significant depth. The significant depth is generally taken as the depth at which the vertical stress is 20% of the load intensity.

According to the above criterion, the depth of exploration should be about 1.5 times the width of the square footing and about 3.0 times the width of the strip footing.

However, if the footings are closely spaced, the whole of the loaded area acts as a raft foundation. In that case, the depth of boring should be at least 1.5 times the width of the entire loaded area.



In the case of pile foundation, the depth of exploration below the tip of bearing piles is kept at least 1.5 times the width of the pile group.

However, in the case of friction piles, the depth of exploration is taken 1.5 times the width of pile group measured from the lower third point.



When the foundations are taken upto rocks, it should be ensured that large boulders are not mistaken as bed rock. The minimum depth of core boring into the bed rock should be 3m to establish it as a rock.

In the case of multi-storeyed buildings, the depth of exploration can be taken from following formula (Sowers and Sowers 1970),

 $D = C(S)^{0.7}$

Here

D = The depth of exploration in mete

C = The constant which is equal to 3 for light steel buildings and narrow concrete bridges. It is equal to 6 for heavy steel buildings and wide concrete buildings.

S = The number of storeys.

If a loose soil or recently deposited soil or weak stratum is encountered, it should be explored thoroughly. Exploration should be carried to a depth at which the net increase in the vertical stress is less than the allowable bearing pressure of the soil.

For two adjacent footings, each of size B x L, spaced at a clear spacing A, IS: 1892-1972 suggests that the minimum depth of boring should be 1.5 B when $A \ge 4B$; and it should be 1.5L when A < 2B. For adjacent rows of such footings, the minimum recommended depth of exploration is 4,5B when A < 2B, it is 3.5B when A > 2B and it is 1.5B when $A \ge 4B$.

For explorations of deep excavations, the depth of excavation below the proposed excavation level should be at least 1.5 times the depth of excavation.

In case of road cuts, the depth of excavation is at least equal to the width of the cut.

In case of road fills, the minimum depth of boring is 2m below the ground surface or equal to the height of fill, whichever is greater.

In case of gravity dams, the minimum depth of boring is twice the height of the dam.

Lateral Extent of Explorations

The lateral extent of exploration and the spacing of bore holes depend mainly on the variation of the strata in the horizontal direction.

The exploration should be extensive so as to reveal major changes in the properties of the sub-surface strata.

For small and less important buildings, even one bore hole or a trial pit in the centre may suffice.

But for compact buildings, covering an area of about 0.4 hectares, there should be at least 5 bore holes, one at the centre and four near the corners.

For large, multi-storeyed buildings, the bore holes should be drilled at all the corners and also at important locations.

the

The spacing between the bore holes is generally kept between 10 to 30 m, depending upon the variation in the subsurface conditions and loading.



For highways, subsurface explorations are usually carried out along the proposed centre line (or) along the proposed ditch line. The spacing of bore holes usually varies between 150 m to 300 m. If the sub-strata is erratic, the spacing may be reduced to even 30 m.

In concrete dams, the spacing of bore holes generally varies between 40 m to 80 m.

| S.No | Nature of Project | Spacing of Bore holes in metres |
|------|----------------------------|---------------------------------|
| 1 | Highways (Subgrade Survey) | 300 to 600 |
| 2 | Earth dam | 30 to 60 |
| 3 | Borrow pits | 30 to 120 |
| 4 | Multi-storeyed buildings | 15 to 30 |
| 5 | Single storey factories | 30 to 90 |

Methods of sub-soil exploration

The methods available for sub-soil exploration may be classified as follows

- 1). Direct methods or Open excavation methods : a). Pits and Trenches
 - b). Drifts and Shafts

2). Semi-direct methods

: Borings

3). Indirect methods

- : a).Soundings (or) Penetration tests and
 - b). Geophysical methods.

Open excavation methods of exploration

In this method of exploration, an open excavation is made to inspect the sub-soil strata. The methods can be divided into two categories 1). Pits and Trenches and 2). Drifts and shafts.

1). Pits and trenches.

Pits and Trenches are excavated at site to inspect the strata. The size of the pit should be sufficient to provide necessary working space. IS : 4453-1967 recommends a clear working space of 1.2 m X 1.2 m at the bottom of the pit.



The depth of pit depends on the requirement of the investigation.

Shallow pits up to a depth of 3 m can be made without providing any lateral support.Fordeeper pits, especially below the ground water table, lateral support in the formofsheeting and bracing system is required.of

As the depth of pit increases, the cost increases rapidly. For depths more than 6 m bore holes are more economical than open pits.

Deep pits should be properly ventilated to prevent the accumulation of dead air. If water is encountered in a pit, it should be suitably dewatered.

Trenches are long shallow pits. As a trench is continuous over a considerable length, it provides exposure along a line.

The trenches are more suitable than pits for exploration on slopes.

Test pits and trenches can be excavated manually or mechanically. Adequate precautions should be taken against possible accidents due to caving of the ground.

2). Drifts and shafts

Drifts are horizontal tunnels made in the hill-side to determine the nature and structure of the geological formation. IS : 4453-1980 recommends that a drift should have the minimum clear dimensions of 1.5 m width and 2.0 m height in hard rock.

In soft rock, an arch roof is more advantages than a flat roof. If the ground is unable to stand of its own, supports have to be provided to carry the load of roof and the sides of the drift.

Drifts are useful for establishing the minimum excavation limits to reach sound rock and for locating faults and shear zones and buried channels in the river sections.

However, drifts are generally expansive. These are used only when other methods do not provide the required information. Drifts are also known as "**adits**".

Shafts are large size vertical holes made in the geological formation. These may be rectangular or circular in section.

The minimum width of a rectangular shaft is 2.4 m and for a circular shaft, the minimum diameter is 2.4 m. In weak ground, the sides of the shaft should be properly supported.

Deep shafts should be properly ventilated. Shafts are used to reach a particular stratum at depth of 4m or more.

Shafts are also used to extend the exploration below the river bed already done by means of tunnels.

Semi direct methods – Borings for soil exploration

When the depth of soil exploration is large, borings are used for exploration.

A vertical bore hole is drilled in the ground to get the required information about the sub-soil strata. Soil samples are taken from the bore holes and tested in the laboratory.

The bore hole may be used for conducting in-situ tests and for locating the water table. Extensometers or pressure meters may also be installed in the bore for the measurement of deformation in the sub-strata.

Depending upon the type of soil and the purpose of boring, the following methods are used for drilling the bore holes.

1). Auger boring 2). Wash boring 3). Rotary boring 4). Percussion boring 5). Core boring.

A few bore holes are drilled during the preliminary investigation. In the detailed investigations, large numbers of bore holes are drilled to thoroughly investigate the sub-soil strata.

1). Auger boring

An auger is a boring tool similar to the one used by a carpenter for boring holes in wood. It consists of a shank with a cross-wise handle for turning and having central tapered feed screw. The augers can be operated either manually or mechanically.



The hand augers used in boring are about 15 to 20 cm in diameter. These are suitable for advancing bore holes upto a depth of 3 to 6 m in soft soils.

The hand auger is attached to the lower end of a pipe of about 18 mm diameter. The pipe is provided with a cross-arm at its top.

The hole is advanced by turning the cross arm manually and at the same time applying the thrust in the down direction.

When the auger is filled with soil, it is taken out. If the bore hole is already driven, another type of auger, known as post-hole auger is used for taking soil samples.

Mechanical augers are driven by power. These are used for making bore holes in hard strata to a greater depth. However for depths greater than 12 m, even mechanical augers becomes inconvenient and other methods of boring are used.

Auger boring is generally used in soils which can stay open without casing or drilling mud. Clays, silts and partially saturated sands can stand unsupported.

For soils which cannot stand unsupported, especially for sandy soils below water table, a casing is normally required.

For such soils, the methods of auger boring become slow and expensive.

Auger boring cannot be used when there are large cobbles, boulders or other obstructions which prevent drilling of bore hole.

Auger borings are particularly useful for subsurface investigations of highways, railways and air fields, where the depth of exploration is small.

The investigations are done quite rapidly and economically by auger boring.

The main disadvantage of auger boring is that the soil samples are highly disturbed. Further, it becomes difficult to locate the exact changes in the soil strata.

2). Wash boring

In wash boring, the bore hole is drilled by first driving a casing, about 2 to 3 m long and then inserting into it a hollow drilled rod with a chisel-shaped chopping bit at its lower end. Water is pumped down the hollow drilled rod, which is known as wash pipe.



The water emerges as a strong jet through a small opening of the chopping bit. The hole is advanced by a combination of chopping action and the jetting action, as the drilling bit and the accompanying water jet disintegrate the soil.

The water and the chopping soil particles rise upward through the annular spaces between the drilled rod and the casing pipe. The return water, also known as wash water, is laden with soil cuttings. It is collected in a tub through a T-shaped pipe fixed at the top of the casing as shown in figure.

The bore hole is further advanced by alternately raising and dropping the chopping bit by a winch. The swivel joint provided at the top of the drilled rod facilitates the turning and twisting of the rod. The process is continued even below the casing till the bore hole begins to cave in. At that stage, the bottom of casing can be extended by providing additional pieces at the top portion.

Sometimes instead of a casing pipe, specially drilling fluids made of suspension or emulsion of fat clays or bentonite combined with some chemical additives are used for supporting the walls of the bore hole.

The wash soil samples collected in the tub do not represent the soil in its true condition. There is complete breakdown of particles by chopping action. There is also mixing of the particles and the loss of fine particles in transportation. The soil samples are of little practical use.

However, some indication about the changes in strata is provided by the reaction of the chopping bit as the hole is advanced. It is also indicated by a change in colour of the wash water.

The wash boring may be used in all types of soils expect those mixed with gravel and boulders. The rocks also cannot be penetrated by this method.

The wash boring is mainly used for advancing a bore hole in the ground. Once the hole has been drilled, a soil sampler is inserted to obtain the soil samples for testing in the laboratory.

The equipment used in wash borings is relatively light and inexpensive. The main disadvantage of this method is that it is slow in stiff and coarse-grained soils.

It cannot be used efficiently in hard soils, rocks and soils containing boulders.

The method is not suitable for talking good quality undisturbed samples above ground water table, as the wash water enters the strata below the bottom of the bore hole and causes an increase in its water content.

3). Rotary drilling

In rotary drilling method, the bore hole is advanced by rotating a hollow drill rod which has a cutting bit at the lower end.

A drilled head is provided at the top of the drill rod. It consists of a rotary mechanism and an arrangement for applying downward pressure.

As the drilling rod is rotated, the cutting bit shears off chips of the material penetrated. A drilling fluid under pressure is introduced through the drilling rod to the bottom of the hole.

The fluid carries the cuttings of the material penetrated from the bottom of the hole to the ground surface through the annular space between the drilling rod and the walls of the bore hole.

The drilling fluid also cools the drilling bit. In case of an uncased hole, the drilling fluid also supports the walls of the bore hole.

When the soil sample is required to be taken, the drilling rod is raised and the drilling bit is replaced by a sampler. Rotary drilling can be used in clay, sand and rocks.

Bore holes of diameter 50 mm to 200 mm can be easily made by this method.

This method is not very well adopted for use in materials containing a large percentage of particles of gravel size and larger.



4). Percussion drilling

The percussion drilling method is used for making bore holes in rocks, boulders and other hard strata.

In this method, a heavy chisel is alternatively lifted and dropped in a vertical hole. The material gets pulverized.

If the point where the chisel strikes is above the water table, water is added to the bore hole. The water forms slurry with the pulverized material, which is removed by a sand pump (or) a bailer at intervals.

Percussion drilling may require a casing. Percussion drilling is also used for drilling of tube wells.

The main advantage of percussion drilling method is that it can be used for all types of materials. It is also particularly useful for drilling holes in glacial tills containing boulders. One of the major disadvantages is that the material at the bottom of the bore hole is disturbed by heavy blows of the chisel.

Due to this, it is not possible to get good quality undisturbed soil samples. Further, this method is more expansive than the other methods.

Moreover it is not possible to detect the minor changes in the properties of the strata penetrated.



5). Core drilling

The core drilling method is used for drilling bore holes and for obtaining rock cores. In this method, a core barrel fitted with a drilling bit is fixed to a hollow drilling rod.

As the drilling rod is rotated, the bit advances and cuts an angular hole around an intact core. The core is then removed from its bottom and it retained by a core lifter and brought to the ground surface.

Water is pumped continuously into the drilling rod to keep the drilling bit cool and to carry the disintegrated material to the ground surface.

The core drilling may be done using either a diamond studded bit or a cutting edge consisting of chilled shot. The diamond drilling is superior to the other type of drilling but it is costlier.

The core barrel may consist of a single tube or a double tube. A double-tube barrel gives a good quality sample of the rock.

<u>UNIT - II</u>

Shallow Foundations:

Bearing Capacity Criteria: Types of foundations and factors to be considered in their location - Bearing capacity – criteria for determination of bearing capacity – factors influencing bearing capacity – analytical methods to determine bearing capacity – Terzaghi's theory - IS Methods.

Settlement Criteria: Safe bearing pressure based on N- value – allowable bearing pressure; safe bearing capacity and settlement from plate load test – Types of foundation settlements and their determination - allowable settlements of structures.

INTRODUCTION

The foundation is the lowest part of the structure which is in contact with the soil or rock lying below and transmits the loads of the structure to it.

The soil which lies below the foundation to which the structural loads are transmitted from the foundation is known as the foundation soil.

The foundations are required for distributing the loads of the structure on a larger area.



Here

 $D_{\rm f}$ = The depth of foundation and

B = The width of foundation

The foundation should be designed such that

- 1). The soil below the foundation does not fail in shear and
- 2). The settlement of the foundation is within the safe limits

General requirements of foundations

Foundations should be constructed to satisfy the following requirements

1). The foundations shall be constructed to sustain the dead and imposed loads and to transmit these loads to the foundation soil in such a way that the pressure on foundation soil will not cause excessive settlement which would impair the stability of the structure.

2). Foundation base should be rigid so that differential settlements are minimized, specially for the case when super-imposed loads are not evenly distributed.

3). Foundation should be taken sufficiently deep to guard the structure against the damage or distress caused by swelling or shrinkage of the foundation soil or sub-soil.

4). Foundation should be so located that its performance may not be affected due to any unexpected future influence.

Foundation Loads

The loads coming on to the foundation of a structure can be classified into four different categories

1). Dead load 2). Live load 3). Wind load 4). Earthquake load

Dead load is the sum of loads comprising the self weight of the structure, and foundation. All permanent loads are included in dead load. These loads can be calculated by finding weights of the cubical contents of different materials used in the structure.

Live load is the movable load on the floor and hence it is variable. It is also known as superimposed load. It includes the weight of persons standing on the floor, weight of the materials temporarily placed on the floor etc.

In case of tall structures, the effect due to wind should be considered. The exposed sides of walls and roofs of structure are subjected to the wind loads. Wind load is based on the wind velocity. The wind loads may be neglected in the design of foundation unless the wind load on the foundation exceeds one-third of the load due to dead load and live load combined.

Earth quake loads must be included in foundation design in countries where seismic shocks are probable. Earthquake motion results in lateral forces that may act on the structure in any horizontal direction. All structures built in the earthquake zones must be designed to resist these lateral forces.

Types of Foundations

Foundations may be broadly classified into two categories

1). Shallow Foundations and 2). Deep Foundations

According to Terzaghi criterion,

1). The foundation is termed as shallow foundation if its depth (D_f) is equal to (or) less than the width (B) of foundation.

2). The foundation is termed as deep foundation if its depth (D_f) is more than the width (B) of foundation

Thus

1). $\frac{D_f}{B} \le 1$ for Shallow foundation and 1). $\frac{D_f}{B} > 1$ for Deep foundation

The classification of Shallow foundations and Deep foundations is as follows



Factors governing the selection of a type of shallow Foundation

The following are the different types of shallow foundations

Spread footing 2). Strip footing 3). Strap Footing 4). Combined footing
Raft or Mat foundation

The selection of a type of shallow foundation is based on the following factors

Soil strength or Bearing capacity of soil 2). Ground conditions 3). Foundation loads
Construction methods 5). Impact on adjacent property

Spread footing is a shallow foundation. The spread footing is selected where the well compacted and stable foundation soil is available near the ground level.Spread footing also called as isolated footing or pad footing or individual footing is provided to support an individual column.

Spread footing is circular, square or rectangular slab of uniform thickness.Spread footing is provided to spread the load over a large area of foundation soil.

Strip footing is a shallow foundation. The strip footing is used support a line of loads such as a load bearing wall. The strip footing could also be used where the line of column positions are so close that individual spread footings would be pointless.

A strap footing usually supports two columns, so it's a special type of combined footing. If the property line exists at or near the edge of an exterior column, a normal isolated footing would be placed eccentrically under this column and it would tend to tilt. This problem may be prevented by connecting this footing with the adjacent interior footing with a strap concrete beam.



The footing used for two columns or more than two columns is called combined footing. The combined footing is mainly two types 1). Rectangular footing and 2). Trapezoidal footing.

The combined footing is provided under the following situations

1). When the columns are located extremely close to each other and their individual footings are overlaying.

2). When the foundation soil is having low bearing capacity and a large area is required under the individual footing.



Raft or mat foundation is shallow foundation. Raft foundation is used to spread the load from a structure over a large area. Raft foundation covers the entire area of te structure.

Raft foundation is often needed on soft or loose soils which have low value of bearing capacity.

The following are the advantages of the raft foundation

- 1). It spread the structural load over a large area
- 2). It provides more structural stability
- 3). It reduces the settlements of the structure
- 4). It offers more resistance to uplift forces
- 5). It distributes the structural loads more evenly onto the foundation soil.

Bearing Capacity of Soils



Foundation Soil





Here

Q = The structural load acting on the foundation and

 σ_2 = The pressure developed in the foundation soil due to structural load



Here

 $\sigma=$ The total pressure on the foundation soil = $\,\sigma_1+\,\sigma_2$

The pressure (σ) which the soil can safely withstand is known as the allowable bearing pressure.

Basic Definitions

The basic definitions relevant to the bearing capacity of soil are as given below.

1). Bearing Capacity

Bearing capacity is defined as the load carrying capacity of the foundation soil which enables it to bear the loads transmitted to it from the structure.

The bearing capacity is usually expressed as the load per unit area.

2). Gross Bearing Capacity

The gross bearing capacity is defined as the bearing capacity inclusive of the surcharge pressure (or) the pressure exerted by the soil standing on the foundation.

3). Net Bearing Capacity

The net bearing capacity is defined as the gross bearing capacity minus the original overburden pressure (or) the surcharge pressure at the foundation level.

 \therefore The net bering capacity = The gross bearing capacity – the overburden pressure

4). Ultimate Bearing Capacity

The ultimate bearing capacity is defined as the maximum load per unit area which the foundation soil can withstand without the occurrence of the shear failure of the foundation soil.

5). Gross Ultimate Bearing Capacity

The gross ultimate bearing capacity is defined as the maximum load per unit area including the surcharge pressure which the foundation soil can withstand without causing the shear failure of the foundation soil.

The gross ultimate bearing capacity is denoted by " qu"

6). Net Ultimate Bearing Capacity

The net ultimate bearing capacity is defined as the maximum load per unit area in excess of the surcharge pressure which the foundation soil can withstand without the causing the shear failure of the foundation soil.

The net ultimate bearing capacity is denoted by " q_{nu} "

Thus

The net ultimate bearing capacity is equal to the gross ultimate bearing capacity minus the surcharge pressure.

| \therefore The net ultimate | = | The gross ultimate | - The surcharge pressure |
|-------------------------------|---|--------------------|--------------------------|
| Bearing Capacity | / | Bearing Capacity | |

$$\therefore q_{nu} = q_u - \gamma D_f$$

Here

 γ =The unit weight of the foundation soil and

 D_f =The depth of foundation

7). Safe Bearing Capacity

The safe bearing capacity is defined as the maximum load per unit area which the foundation soil can carry safely without the risk of shear failure.

The safe bearing capacity is obtained by dividing the ultimate bearing capacity with a suitable factor of safety.

The safe bearing capacity =
$$\frac{\text{The ultimate bearing capacity}}{\text{The factor of safety}}$$

The factor of safety for foundations may range from 2 to 5.

8). Net safe Bearing Capacity (qns)

The net safe bearing capacity is defined as the maximum load per unit area in excess of the surcharge pressure which the foundation soil can carry safely without the risk of shear failure.

The net safe bearing capacity is obtained by dividing the net ultimate bearing capcity with a suitable factor of safety.

The net safe bearing capacity = $q_{ns} = \frac{\text{The net ultimate bearing capacity}}{\text{The factor of safety}}$

$$\therefore q_{\rm ns} = \frac{q_{\rm nu}}{F}$$

9). Gross Safe Bearing Capacity (Qs)

The gross safe bearing capacity is defined as the maximum load per unit area including the surcharge pressure which the foundation soil can withstand without the risk of shear failure of the foundation soil.

The gross safe bearing capacity is denoted by " q_s"

The gross safe bearing capacity is equal to the net safe bearing capacity plus the original surcharge pressure.

$$\therefore q_{s} = q_{ns} + \gamma D_{f}$$
$$\therefore q_{s} = \frac{q_{nu}}{F} + \gamma D_{f}$$

In the design of foundations, the gross safe bearing capacity of the foundation soil (q_s) of the foundation soil is considered.

Factors affecting the Bearing capacity of Foundation Soil

The following factors will affect the bearing capacity of the foundation soil.

1). Nature of the soil and its physical and engineering properties.

2). Nature of foundation such as the size, shape, depth below the ground level etc.

3). Total and differential settlements that the structure can withstand without the failure.

4). Location of the ground water table relative to the level of the foundation

5). Initial stresses in the soil below the ground level

Criteria for the Determination of Bearing Capacity

The determination of bearing capacity of foundation soil is based on the following two criterion

1). The shear failure of the foundation soil shall not occur – shear strength criteria

2). The settlement of the foundation soil must be limited to safe and acceptable magnitude – settlement criteria

The shear strength criteria and the settlement criteria are independent.

The design value of safe bearing capacity would be the smaller of the two values obtained from these two criteria.

Methods to Determine the Bearing Capacity

The following methods are adopted to determine the bearing capacity of the foundation soil

1). Analytical methods 2). Plate bearing tests 3). Penetration tests

4). Laboratory tests and 5). Bearing capacity tables in various codes

The number of analytical method given by Rankine, Prandtl, Terzaghi, Meyerhof, Hansen, Vesic, Skemptonetc are used to determine the bearing capacity of the foundation soil.

The plate bearing tests like the plate load test are conducted in the field to determine the bearing capacity of foundation soil.

Penetration tests are conducted in the field by using the penetrometers to measure the resistance of the soil to penetration. This resistance can be correlated to the bearing capacity of the soil.

Simple laboratory tests may be useful to determine the bearing capacity of pure clays.

Rankine's Analysis

Rankine (1885) considered the equilibrium condition of two soil elements, the first soil element is just below the foundation and the second soil element is just outside the base level of the foundation as shown in the figure.



The soil elements are subjected to vertical principal stress and horizontal principal stress.

For the first soil element, the vertical stress (σ_1) is the major principal stress and the horizontal stress (σ_3) is the minor principal stress.

When the foundation pressure approaches the ultimate bearing capacity (q_u) , the first soil element attains the stage of plastic equilibrium and it is in the verge of failure.

At the state of plastic equilibrium, the major principal stress (σ_1) acting on the first soil element is equal to the ultimate bearing capacity (q_u) and the first soil element is in active state condition.

$$\cdot \sigma_1 = q_u$$

Now, the minor horizontal principal stress (σ_3) acting on the first element is given by

$$\sigma_3 = K_a \sigma_1 But$$
 $K_a = The coefficient of active lateral earth pressure = \left(\frac{1-\sin\phi}{1+\sin\phi}\right)$

$$\therefore \sigma_3 = \left(\frac{1-\sin\phi}{1+\sin\phi}\right) q_u - \dots \dots (1)$$

For the second soil element, the vertical stress (σ_3) is the minor principal stress and the horizontal stress (σ_1) is the major principal stress.

The vertical minor principal stress (σ_3) acting on the second soil element due to the weight of the over burden soil is given by

$$\sigma_3 = \gamma D_f$$

The second soil element is in passive state condition.

Now, the passive major horizontal principal stress (σ_1) acting on the second element is given by

$$\sigma_{1} = K_{p}\sigma_{3}But \quad K_{p} = \text{The coefficient of passive lateral earth pressure} = \left(\frac{1+\sin\phi}{1-\sin\phi}\right)$$

$$\therefore \sigma_{1} = \left(\frac{1+\sin\phi}{1-\sin\phi}\right) \gamma D_{f} - \dots - (2)$$

Since both elements are at the same depth from the ground level

The horizontal minor principal stress (σ_3) (σ_1) acting on the second element. = The horizontal major principal stress acting on the first element

Now, from equations (1) and (2) we get

$$\begin{pmatrix} \frac{1-\sin\phi}{1+\sin\phi} \end{pmatrix} q_{u} = \begin{pmatrix} \frac{1+\sin\phi}{1-\sin\phi} \end{pmatrix} \gamma D_{f}$$

$$\therefore q_{u} = \begin{pmatrix} \frac{1+\sin\phi}{1-\sin\phi} \end{pmatrix}^{2} \gamma D_{f}$$
(3)

The above equation gives the approximate value of the ultimate bearing capacity of the foundation soil.

Rankine did not consider the cohesion (C) of the soil.

Prandtl's Analysis

Prandtl developed a theory for the penetration of punches into metals.

This theory has been used to determine the ultimate bearing capacity of the soil.

Prandtl's analysis is based on the assumption that the strip footing placed on the ground surface sinks vertically downwards into the soil at failure.

The figure shows the failure zones developed below the foundation.

The soil in the wedge shaped zone -I immediately below the foundation is subjected to compressive stresses.

When the foundation starts sinking, soil in zone -I exerts pressure on the soil in the side zones II and III.

The soil in zone - II is assumed to be in plastic equilibrium condition. The soil in zone - II pushes the soil in zone - III in upward direction.

Using the theory of plasticity, Prandtl developed an expression for the bearing capacity of the strip foundation by assuming the curved part of the slip surface of zone - II as the shape of logarithmic spiral.



Prandtl's analysis gave the following expression for the ultimate bearing capacity (qu) the soil

 $q_u = (\pi + 2) C = 5.14 C ----- (1)$

Here C = The cohesion of the soil.

For purely cohesive soil ($\emptyset = 0$) the logarithmic spiral becomes as a circular arc.

The above equation indicates that the ultimate bearing capacity of cohesive soil (q_u) is independent of the width of the foundation (B).

Prandtl's theory is applicable for the foundation at the ground surface.

For a strip foundation on the cohesive soil at a depth ' D_f ' below the groundsurface the ultimate bearing capacity is given by

 $q_u = 5.14 \text{ C} + \gamma D_f$ ------ (2)

Hogentogler and Terzaghi's Analysis

Hogentogler and Terzaghi (1929) approximated the actual failure surfaces in the foundation soil below the foundation with a set of straight lines at the plastic equilibrium condition for a long strip foundation of width 'B' as shown in the figure.



At the time of failure, the foundation exerts a pressure " q_u " equal to the ultimate bearing capacity of the foundation soil.

The soil in zone – I immediately below the foundation is in compression.

The soil in zone – I can fail only when the soil in the adjacent zone – II also fails.

The approximate value of the bearing capacity of foundation soil can be obtained by considering the stresses at the mid heights of the two failure zones.

The height of the failure zone – II is $\frac{B}{2} \tan \alpha$

Here $\alpha = 45^0 + \frac{\phi}{2}$

The overburden pressure or the surcharge pressure on zone – II = γD_f

Consider the zone – II

At the mid height of zone II

 σ_1 =The horizontal major principal stress and

 σ_3 =The vertical minor principal stress

Here

The minor principal stress = $\sigma_3 = \left(D_f + \frac{1}{2}\frac{B}{2}\tan\alpha\right)\gamma$

$$\therefore \sigma_3 = \gamma D_f + \gamma \frac{B}{4} \tan \alpha - \dots$$
 (1)

If the foundation soil is cohesive then in passive condition

The major principal stress ' σ_1 ' is given by

$$\sigma_{1} = \sigma_{3} \tan^{2} \alpha + 2c \tan \alpha$$
$$\therefore \sigma_{1} = \left(\gamma D_{f} + \gamma \frac{B}{4} \tan \alpha\right) \tan^{2} \alpha + 2C \tan \alpha - \dots (2)$$

Consider the zone – I

At the mid height of zone I

 σ_1 =The vertical major principal stress and

 σ_3 =The horizontal minor principal stress

Here

The vertical major principal stress (σ_1) in zone – I is given by

$$\sigma_1 = q_u + \gamma \frac{B}{4} \tan \alpha - \dots \quad (3)$$

The horizontal minor principal stress (σ_3) = The horizontal major principal stress (σ_1) acting in zone - II

$$\therefore \sigma_3 = \left(\gamma D_f + \gamma \frac{B}{4} \tan \alpha\right) \tan^2 \alpha + 2C \tan \alpha - \dots$$
(4)

If the foundation soil is cohesive then in active condition

 $\sigma_3 = \sigma_1 \cot^2 \alpha - 2C \cot \alpha - \dots$ (5)

Substituting the equations (3) and (4) in equation (5) we get

$$\left(\gamma D_{f} + \gamma \frac{B}{4} \tan \alpha\right) \tan^{2} \alpha + 2C \tan \alpha = \left(q_{u} + \gamma \frac{B}{4} \tan \alpha\right) \cot^{2} \alpha - 2C \cot \alpha$$

From the above equation, we get

$$q_u = \gamma D_{\rm f} \tan^4 \alpha + \gamma \frac{B}{4} (\tan^5 \alpha - \tan \alpha) + 2C (\tan^3 \alpha + \tan \alpha) - \dots$$
(6)
The equation (6) gives the general equation for the ultimate bearing capacity (q_u) of the foundation soil.

The equation (6) is applicable to both cohesive and cohesionless soils.

a). For cohesionless soil

C = 0

$$\therefore q_u = \gamma D_{\rm f} \tan^4 \alpha + \gamma \frac{{\rm B}}{4} (\tan^5 \alpha - \tan \alpha)$$

b). For pure cohesive soils $\emptyset = 0$

$$\therefore \alpha = 45^{\circ} + \frac{\emptyset}{2} = 45^{\circ} \qquad \therefore \tan \alpha = 1$$

$$q_u = \gamma D_f + 4C$$

Now

The net ultimate bearing capacity of the soil = $q_{nu} = q_u - \gamma D_f$

 $q_{nu} = 4C$

Terzaghi's Analysis

Terzaghi (1943) gave a theory for the bearing capacity of foundation soil under a strip foundation.Terzaghi's analysis is an extension and improved modification of Prandtl's analysis.

Terzaghi's analysis is based on the following assumptions

1). The base of the foundation is rough (In Prandtl's analysis the foundation base is smooth which is not practical).

2). The foundation is located at a depth 'D_f' below the ground level such that the depth 'D_f' is less than or equal to the width (B) of the foundation, that is $D_f < B$. Thus the foundation is a shallow foundation.

3). The shear strength of soil above the base of the foundation is neglected. The soil above the base of foundation is replaced by an equivalent surcharge pressure " γD_{f} ".

4). The load on the foundation is vertical and it is uniformly distributed.

5). The strip foundation is long that is $\frac{L}{B}$ ratio is infinite, where 'L' is the length of the foundation and 'B' is the width of the foundation.

6). The shear strength of the soil is governed by the Mohr-Coulomb equation.

According to the Terzaghi's analysis, the loaded foundation soil fails along the failure surface "GFCDE" as shown in the figure.



The region in the failure surface "GFCDE" is separated into five zones

1). One zone – I "ABC"2). Two Zone – II "BCD" and "ACG"3). Two zone – III "BDE" and "AGF"

The zone – I "ABC" is wedge shaped located immediately below the foundation. It is assumed that its boundaries 'AC' and 'BC' are plane surfaces and |CAB| and |CBA| are equal to the angle of shearing resistance " \emptyset " of the soil.



Here

 $AC = BC = \frac{B}{2\cos\phi} and H = \frac{B}{2} \tan\phi$

The soil in zone -I is prevented from undergoing any lateral yield by the friction and adhesion between the soil and the base of the foundation.

Thus the soil in zone - I remains in a state of elastic equilibrium and it behaves as if it was a part of foundation.

Zones – II, "BCD" and "ACF" are called the radial shear zones. In these zones the boundaries 'AC', 'AF', 'BC' and 'BD' are the plane surfaces. The boundaries 'CF' and 'CD' are the arcs of a logarithmic spiral.

Z0nes – III, "AFG" and "BDE" are called the zones of linear shear. These are Rankine's passive zones and boundaries are making an angle of $\left(45^0 + \frac{\emptyset}{2}\right)$ with the horizontal.

In Terzaghi's analysis it is assumed that the failure zones do not extend above the horizontal plane passing through the base of the foundation.

The application of load intensity " \mathbf{q}_{u} " on the foundation tends to push the wedge of the soil "ABC" into the ground with lateral displacement of zones II and III.

But this lateral displacement is resisted by the zones II and III and hence forces will act on the planes 'AC' and 'BC' of the soil wedge "ABC".

The forces acting on the planes 'AC' and 'BC' in zone - I are

1). The resultant passive earth pressure ' P_p ' and

2). The force due to the unit cohesion (c) of the soil.



The resultant passive earth pressure (P_p) will acts vertically upward making an angle ' \emptyset ' with the normal to the surfaces 'AC' and 'BC'.

The force due to unit cohesion (c) will acts along the surfaces 'AC' and 'BC'.

The length of surfaces 'AC' or 'BC' = $\frac{B}{2\cos\phi}$

At the instant of failure, the downward forces and the upward forces acting on the wedge "ABC" must be balanced.

By considering the unit length of the shallow foundation,

The downward forces acting on the soil wedge "ABC" are

- 1). The total load from the foundation = $q_u \times B \times 1 = q_u B$
- 2). The weight of soil wedge 'ABC' = W = γ x Volume of wedge 'ABC' = γ x V

$$= \gamma x Ax L = \gamma x \frac{1}{2} x B x H X$$
$$= \gamma x \frac{1}{2} x B x \frac{1}{2} x B x \frac{1}{2} x B x \frac{1}{2} \tan \phi x 1$$
$$\therefore W = \frac{1}{4} \gamma B^{2} \tan \phi$$

1

: The total downward force acting on zone – I = $q_u B + \frac{1}{4}\gamma B^2 \tan \emptyset$

The upward forces acting on the soil wedge "ABC" are

1). The resultant passive earth pressure acting on each of surfaces 'AC' and 'BC' = $2P_P$ and

2).The vertical component of the force due to cohesion acting one each of surfaces 'AC' and 'BC'

Here

The total vertical cohesive force = $c \times AC \times \sin \phi + c \times BC \times \sin \phi = 2 \times c \times AC \times \sin \phi$

$$= 2 x c x \frac{B}{2 \cos \emptyset} x \sin \emptyset$$

= B x c xtan Ø

: The total upward force acting on zone – I = 2 P_P +B x c xtan Ø

At the condition of failure

The total downward force acting on Zone - I = The total upward force acting on zone - I

$$\therefore q_u B + \frac{1}{4}\gamma B^2 \tan \emptyset = 2 P_P + B x c x \tan \emptyset$$
$$\therefore q_u B = 2 P_P + B x c x \tan \emptyset - \frac{1}{4}\gamma B^2 \tan \emptyset \quad ------ \quad (1)$$

Consider the Zone – II and Zone – III in the foundation soil as shown in the figure.



The resultant passive earth pressure (P_p) acting in Zone – II and Zone – III has the following three components

1). $P_{P\gamma}$ produced by the weight of foundation soil in the portion "BCDE"

2). P_{PC} Produced by the soil cohesion 'c' acting on the surface 'BC'

3). P_{Pq} produced by the surcharge weight of the soil on the surface 'BE'.

 $\therefore P_{P} = P_{P\gamma} + P_{Pc} + P_{Pq}$ ------(2)

Substituting the equation (2) in equation (1) we get

$$\therefore q_{u} B = 2 \left[P_{P\gamma} + P_{Pc} + P_{Pq} \right] + B x c x tan \emptyset - \frac{1}{4} \gamma B^{2} tan \emptyset$$

$$\therefore q_{u} B = \left(2P_{P\gamma} - \frac{1}{4} \gamma B^{2} tan \emptyset \right) + \left(2P_{Pc} + B x c x tan \emptyset \right) + 2 P_{Pq}$$
According to Terzaghi's Analysis
$$1). \ 2P_{P\gamma} - \frac{1}{4} \gamma B^{2} tan \emptyset = \frac{1}{2} B \gamma B N_{\gamma}$$

$$2). \ 2P_{Pc} + B x c x tan \emptyset = B c N_{C} \text{ and}$$

$$3). \ 2 P_{Pq} = B \gamma D_{f} N_{q}$$

$$\therefore q_{u} B = \frac{1}{2} B \gamma B N_{\gamma} + B c N_{C} + B \gamma D_{f} N_{q}$$

$$\therefore q_{u} = \frac{1}{2} \gamma B N_{\gamma} + c N_{C} + \gamma D_{f} N_{q}$$
But $\gamma D_{f} = \text{The surcharge pressure} = q$

$$\therefore q_{u} = c N_{C} + q N_{q} + 0.5 \gamma B N_{\gamma} - \dots$$

$$(3)$$

The above equation is known as Terzaghi's bearing capacity equation for a strip footing.

In terms of the effective cohesion (c') the above equation can be written as

$$\therefore q_u = c' N_C + q N_q + 0.5 \gamma B N_{\gamma}$$

In the above equations the terms 'N_C', 'N_q' and N_{γ} are called the bearing capacity factors. They are dimensionless numbers and depends on the angle of shearing resistance ' \emptyset ' of the soil.

q= The effective surcharge at the base level of foundation = $\gamma \, D_f$

Square and Circular Footings

The Terzaghi's bearing capacity equation for a strip footing is given by

 $\therefore q_u = c N_C + q N_q + 0.5 \gamma B N_{\gamma} = c N_C + \gamma D_f N_q + 0.5 \gamma B N_{\gamma}$

Based on the experimental results, Terzaghi gave the following bearing capacity equations for square and circular footing

a). Square footing

 $\therefore q_u = 1.2 \text{ c } N_C + \gamma D_f N_q + 0.4 \gamma B N_{\gamma}$

b). Circular footing

 $\therefore q_u = 1.2 \ c \ N_C + \gamma \ D_f \ N_q + \quad 0.3 \ \gamma \ D \ N_{\gamma} \label{eq:quarter}$

D = The diameter of the footing

Effect of water table on bearing capacity

The Terzaghi's bearing capacity equation for a strip footing is given by

 $\therefore q_u = c \ N_C + \ q \ N_q + \ 0.5 \ \gamma \ B \ N_\gamma = \ c \ N_C + \gamma \ D_f \ N_q + \ 0.5 \ \gamma \ B \ N_\gamma$

This equation is developed based on the assumption that the water table is located at a greater depth below the foundation.But if the water table is located close to the foundation then the above bearing capacity equation needs the modification as explained below.

1). Water table located above the base of the foundation



Let

 $D_{\rm f}$ = The depth of foundation from the ground level

 D_w = The depth of water table form the ground level

a = The height of water table above the base of foundation = $D_f - D_w$

 γ = The unit weight of soil above the foundation

 γ' = The submerged unit weight of soil below the foundation

Now, the surcharge pressure 'q'acting at the base level of the foundation is given by

$$q = \gamma \ D_w + \gamma' \ a = \gamma D_w + \ \gamma' (D_f - \ D_w) = \ \gamma' D_f + \ D_w (\gamma - \ \gamma')$$

In this case, the unit weight of soil in the third term of Terzaghi's bearing capacity equation is taken as the submerged unit weight of the soil (γ').Hence the Terzaghi's bearing capacity equation for this condition of water table changes as

 $\therefore q_u = c \ N_C + \ \left(\gamma' D_f + \ D_w (\gamma - \ \gamma') \right) \ N_q + \ 0.5 \ \gamma' \ B \ N_\gamma$

B W.T.

2). Water table located at a depth 'b' below the base of the foundation

Case 01 : If $\mathbf{b} \ge \mathbf{B}$ then there is no change in the Terzaghi's bearing capacity equation

Case 02 : If $\mathbf{b} < \mathbf{B}$ that is the water table is located at the level of the base of footing or below it, then the second term or the surcharge term ($\mathbf{q} \mathbf{N}_{q}$) is not affected.

However the unit weight of soil in the third term $(0.5 \gamma B N_{\gamma})$ is modified as below

$$\gamma^* = \gamma' + \frac{b}{B}(\gamma - \gamma')$$

Now, the Terzaghi's equation for bearing capacity of soil is given by

$$\therefore q_u = c \ N_C + \ q \ N_q + \ 0.5 \ \gamma^* \ B \ N_\gamma = \ c \ N_C + \gamma \ D_f \ N_q + \ 0.5 \ \left(\gamma' + \frac{b}{B}(\gamma - \ \gamma')\right) \ B \ N_\gamma$$

Meyerhof's Bearing Capacity Theory

Meyerhof (1951) gave a general theory for the bearing capacity of foundation soil under a strip footing which is applicable to both shallow as well as deep foundations.

The failure surfaces assumed by Meyerhof are as shown in the figure.

Meyerhof considers the failure mechanism similar to that assumed by Terzaghi, but extended the failure surfaces above the base of the foundation.

Thus the shear strength of soil above the base of the footing was also considered in the analysis.

The curved failure surfaces in the zone of radial shear were assumed to be the logarithmic spirals.



The significance of different zones in Meyerhof's bearing capacity theory are

1). Zone – I "ABC" which is elastic zone but the angles which the inclined surfaces "AC" and "BC" make with horizontal varies between \emptyset and $\left(45^0 + \frac{\emptyset}{2}\right)$

2). Zone - II "ACG" and "BCD" which are the radial shear zones and

3). Zone – III "AGHI" and "BDEF" which is the mixed shear zone in which the shear varies from radial shear to plane shear.

The surface "AH" or "BE" is known as the equivalent free surface which is making an angle " β " with the horizontal.

<u>UNIT - 03</u>

Pile Foundations:

Types of piles – Load carrying capacity of piles based on static pile formulae – Dynamic pile formulae– Pile load tests - Load carrying capacity of pile groups in sands and clays.

Well Foundations:

Types – Different shapes of well – Components of well – functions – forces acting on well foundations - Design Criteria – Determination of staining thickness and plug – construction and Sinking of wells – Tilts and Shifts

Pile Foundations

When the soil near the ground surface is not capable of supporting a structure, deep foundation is required to transfer the loads to deeper hard strata.

The most common types of deep foundations are Pile foundations, Pier foundations and Well foundations.

The pile is made of steel, concrete or wood.

The pile is either driven into the soil or formed in-situ by excavating a bore hole into the soil and filling it with concrete.

The **Pile** greater than 0.6m diameter is generally termed as **Pier**.

Necessity of Pile Foundation

Pile foundations are used in the following conditions

1). When the soil strata just below the ground surface is highly compressible and very weak to support the load transmitted by the structure.

2). Pile foundation is required for the transmission of structural loads through deep water to a firm stratum.

3). Pile foundation is required when the soil conditions are such that erosion or scour of soil may occur from underneath a shallow foundation.

4). In case of expansive soils such as black cotton soil, which swell or shrink as water content changes, the pile foundation is required to transfer the loads below the active zone.

5). Pile foundation is required for some structures such as transmission towers and off-shore platforms which are subjected to uplift effect.

Classification of Piles

The piles are classified

- 1). Based on the material used
- 2). Based on the mode of transfer of load
- 3). Based on the method of construction or installation
- 4). Based on the usage and

5). Based on the displacement of soil

1). Classification based on material used

According to the materials used the piles are classified as

a). Steel piles b). Concrete piles c). Timber piles and d). Composite piles

Steel piles are generally either in the form of thick pipes or rolled steel H- sections. The steel piles are provided with a driving shoe at the lower end. Epoxy coatings are applied on the surface of steel piles to reduce the corrosion of the pile.

Cement concrete is used in the construction of concrete piles. Concrete piles are either precast or cast-insitu. The cast-insitu concrete piles are either cased or uncased.

Timber piles are made from tree trunks after proper trimming. The timber used should be straight, sound and free from defects. Steel shoes are provided at the ends of timber pile to prevent damage during the driving process. Timber piles should not be used in marine environment where these piles are attacked by various micro-organisms.

Composite pile is made of two different materials. A composite pile may consist of the lower portion steel and the upper portion of cst-insitu concrete. As it is difficult to provide a proper junction between two different materials, composite piles are rarely used in practice.

2). Classification based on mode of transfer of loads

According to the mode of transfer of loads, the piles are classified as

a). End-bearing piles b). Friction piles and c). Combined end-bearing and friction piles

The end-bearing piles transmit the loads through the pile tip to a suitable hard stratum, passing the soft soil stratum.

The ultimate load carrying capacity (Q_u) of the end bearing pile depends upon the bearing capacity of the hard stratum. The ultimate load (Q_u) carries by the end bearing pile is equal to the load carried by the pile point (Q_p)



The friction piles are used when the hard stratum does not exist at a reasonable depth from the ground surface.

The friction piles do not reach the hard stratum.

The friction piles transfer the load through skin friction between the embedded surface of the pile and the surrounding soil.

The ultimate load (Q_u) carried by the friction pile is equal to the load transferred by skin friction (Q_s) .

The friction piles are also known as the floating piles.



The combined end bearing and friction piles transfer the loads by a combination of end bearing at the bottom of the pile and friction along the surface of the pile shaft.

The ultimate load carried by the pile (Q_u) is equal to the sum of the load carried by the pile point (Q_p) and the load carried by the skin friction (Q_s) .

$$\therefore Q_U = Q_P + Q_S$$

3). Classification based on method of installation

According to the method of installation (or) construction, the piles are classified as

a). Driven piles :

These piles are driven into the soil by applying blows of heavy hammer on their top

b). Driven and Cast-insitu piles

These piles are formed by driving a casing pipe into the soil. The casing pipe is later filled with concrete.

c). Bored and Cast-insitu piles

These piles are formed by excavating a hole into the ground and then filling it with concrete

d). Jacked piles

These piles are jacked into the soil by applying a downward force with the help of a hydraulic jack.

4). Classification based on usage

According to the usage the piles are classified as

a). Load bearing piles

These piles are used to transfer the loads of the structure to a suitable hard stratum either by end bearing or by skin friction or by both

b). Compaction piles

These piles are driven into the loose granular soils to increase the relative density. The bearing capacity of the soil is increased due to densification caused by vibration.

c). Tension piles

These piles are in tension. These piles are used to anchor down the structures subjected to hydrostatic uplift forces or overturning forces.

5). Classification based on displacement of soil

According to the volume of soil displaced during installation, the piles are classified as

a). Displacement piles

All driven piles are displacement piles. The soil is displaced laterally when the pile is driven into it. Precast concrete piles and closed end pipe piles are highly displacement piles. Steel H-piles are low displacement piles.

b). Non – displacement piles

The bored piles are non-displacement piles. The soil is not subjected to displacement during the drilling of bore hole.

Load Carrying Capacity of Piles

The methods for estimating the load carrying capacity of a pile foundation are grouped into the following four categories

1). Static methods

According to the static methods

The ultimate load capacity (Q_U) of an individual pile is given by

 $Q_U = Q_P + Q_S$

Here

- Q_U = The ultimate failure load of pile foundation
- Q_P = The point or the base resistance of pile
- Q_S = The shaft resistance developed due to friction between the soil and the pile

The static methods provide a reasonable estimation of the pile capacity.

2). Dynamic formulae

The ultimate load carrying capacity of a driven pile can be estimated from the Dynamic formulae. The ultimate carrying capacity of a driven pile in certain types of soils is related to the resistance against the penetration developed during the driving operation.

The dynamic formulae are based on the assumption that the kinetic energy delivered by the hammer during driving operation is equal to the work done on the pile.

3). In-situ penetration tests

The ultimate load capacity of a pile can be determined from the results of in-situ standard penetration test.

Empirical formulae are used to determine the point resistance (Q_P) and the shaft resistance (Q_P) from the standard penetration number (N).Cone penetration test is also used to estimate the pile capacity.

4). Pile load test



The pile load test is most reliable method to determine the ultimate load capacity of a pile. The pile load test, the test pile is driven and loaded up to failure.

The pile capacity is related to either the load at failure or the load at which the settlement do not exceed the permissible limits.

Static Methods for Driven Pile in Sand

According to the static methods

The ultimate load carrying capacity (Q_U) of a single pile driven into sand is given by

$$Q_U = Q_P + Q_S$$

Here

 Q_U = The ultimate failure load of pile foundation

 Q_P = The tip resistance of pile = $q_P A_P$

 Q_S = The shaft resistance developed due to friction between the soil and the pile = $f_S A_S$

$$\therefore Q_{\rm U} = q_{\rm P}A_{\rm P} + f_{\rm S}A_{\rm S}$$

In the above equation

 q_P = The ultimate bearing capacity of the soil at the pile tip

 A_P = The area of the pile tip

 f_S = The average unit skin friction between the sand and the pile surface and

 A_{S} = The effective surface area of the pile in contact with the sand

a). Method of determination of "Qp"

The tip resistance of pile = $Q_P = q_P A_P$

 q_P = The ultimate bearing capacity of the soil at the pile tip and

 A_P = The area of the pile tip

The ultimate bearing capacity (q_P) at the pile tip in sandy soils is given by

$$q_P = \bar{q}N_q + 0.4 \gamma B N_\gamma$$

Here

 $\overline{\mathbf{q}}$ = The effective vertical pressure at the pile tip.

 $\mathbf{B} = \mathbf{Pile}$ tip width or diameter

 γ = The unit weight of soil in the zone of the pile tip

 N_q and N_γ = The bearing capacity factors for deep foundation

In driven piles the second term " $0.4 \gamma B N_{\gamma}$ " is small and hence it is neglected.

Thus for driven piles

 $q_P = \bar{q}N_q$

In case of driven piles, the effective vertical pressure (\bar{q}) at the tip of the pile will increase with the depth (D) only up to certain depth known as the critical depth (D_C). After the critical depth the effective vertical pressure remains constant.

The critical depth (D_C) depends on the angle of shearing resistance of the soil (\emptyset') and the width of the pile tip (B).

The critical depth (D_C) is taken as "10 B" in loose sand and "20 B" in dense sand.

N_q is the bearing capacity factor

 \therefore The tip resistance of pile = $Q_P = \overline{q}N_qA_P$

b). Method of determination of "Qs"

The shaft resistance due to friction (Qs) on pile foundation is given by

 $Q_S = f_S A_S$

Here

 f_S = The unit skin friction between the sand and the pile surface

 A_S = The surface area of the pile = P x D

P = The perimeter of the pile and

D = The depth of pile



The unit skin friction (f_S) for a straight – sided pile depends upon the soil pressure acting normal to the pile surface and the coefficient of friction between the sand and the pile material.

The soil pressure acting normal to the vertical straight – sided pile surface is the horizontal pressure (σ_h)

The relation between the horizontal pressure (σ_h) and the effective vertical pressure $(\overline{\sigma_V})$ is given by

$$\sigma_{\rm h} = K \overline{\sigma_{\rm V}}$$

Here

K = The lateral earth pressure coefficient and

 $\overline{\sigma_V}$ = The effective vertical pressure

The unit skin friction (f_S) acting on the pile at any depth is given by

 $f_S=\,\sigma_h\,tan\,\delta$

 $\therefore f_S = K \overline{\sigma_V} \tan \delta$

Here

 $tan \delta$ = The coefficient of friction between the sand and the pile materials



Static Methods for bored Pile in clay

According to the static methods

The ultimate load carrying capacity (Q_U) of a single bored pile in clay soil is given by

$$Q_U = c N_c A_P + c_a A_S = c N_c A_P + \alpha c A_S$$

Here

The value of " α " depends upon the pile type and the method of drilling the bore hole.

For straight piles with bore hole drilled in dry soil condition the value of ' α ' is equal to 0.5

When the bore hole is drilled with Bentonite slurry then the value of ' α ' is equal to 0.3

For calculating the surface area of the straight pile, the upper 1.5 m length and the lower 1.5 m is neglected.

Dynamic Formulae

The dynamic formulae are based on the assumption that the kinetic energy delivered by the hammer during the driving operation is equal to the work done on the pile.

The dynamic formulae are used to determine the ultimate load capacity of the driven piles.

According to the dynamic formulae

 $\eta_{h\,W\,h=R\,S}$

Here

W = The weight of hammer in 'kN;

h = The height of hammer drop in 'cm'

 η_h = The efficiency of the hammer

R = The pile resistance in 'kN'

 $= Q_{\rm U} = \text{The ultimate load capacity of the} \\ \text{pile}$

S = the penetration for one hammer blow

The following dynamic formulae are used in pile foundations

- 1). Engineering News Record formula
- 2). Hiley formula and
- 3). Danish formula

1). Engineering News Record Formula



According to the Engineering News Record Formula, the ultimate load capacity (Q_U) of a driven pile is given by

$$Q_{U} = \frac{W h \eta_{h}}{S + C} = \frac{E_{n} \eta_{h}}{S + C}$$

Here

W = The weight of hammer in 'kN'

h = The height of hammer drop in 'cm'

 $E_n = W.h =$ The rated energy of hammer in kN-cm

S = The penetration of pile per one hammer blow in 'cm' and

C = Constant

= 2.54 cm for drop hammer and

= 0.254 cm for steam hammer

The efficiency of the hammer ' η_h ' of single acting drop hammer is between 0.7 to 0.9

The efficiency of the hammer ' η_h ' of double acting drop hammer is between 0.75 to 0.85

The efficiency of the hammer ' η_h ' of diesel hammer is between 0.8 to 0.9

The factor of safety of '6' is usually recommended

According to the Modified Engineering News Formula

The ultimate load capacity (Q_U) of a driven pile is given by

$$Q_{U} = \frac{W h \eta_{h}}{S + C} \left(\frac{W + e^{2}P}{W + P} \right)$$

Here

P = The weight of pile in 'kN' and

e = The coefficient of restitution

The efficiency of the hammer ' η_h ' of single acting drop hammer is between 0.75 to 1.0

The efficiency of the hammer ' η_h ' of double acting drop hammer is between 0.75 to 0.85

The efficiency of the hammer ' η_h ' of diesel hammer is between 0.85 to 1.0

The efficiency of the hammer ' η_h ' of differential hammer is between 0.85.

The values of coefficient of restitution are as follows

| Type of Pile | Coefficient of restitution 'e' |
|--|--------------------------------|
| | |
| Broomed Timber Pile | 0.0 |
| | |
| Good Timber Pile | 0.2 |
| | |
| Driving cap with timber dolly on steel pile | 0.3 |
| | |
| Driving cap with plastic dolly on steel pile | 0.5 |
| | |
| Helmet with composite plastic dolly and packing on R.C.C | 0.4 |
| pile | |

2). Hiley Formula

According to Hiley Formula, the ultimate load capacity (Q_U) of a driven pile is given by

$$Q_{U} = \frac{W h \eta_{b} \eta_{h}}{S + \frac{C}{2}} = \frac{E_{n} \eta_{b} \eta_{h}}{S + \frac{C}{2}}$$

Here

W = The weight of hammer in 'kN'

h = The height of free fall of the ram (or) hammer in 'cm'

 $E_n = W.h =$ The rated energy of hammer in kN-cm

S = The final set (or) the penetration of pile per one hammer blow in 'cm' and

 $\eta_h =$ The hammer efficiency

 η_b = The efficiency of hammer blow

- C = The sum of temporary elastic compression of the pile, dolly, packings and ground $= C_1 + C_2 + C_3$
- C₁ = The temporary compression of dolly and packing = $1.77 \frac{R}{A}$ When the driving is without dolly = $9.05 \frac{R}{A}$ When the driving is with short dolly

 C_2 = The temporary compression of pile = 0.657 $\frac{RD}{A}$

 C_3 = The temporary compression of ground = $3.55\frac{R}{A}$

D = The length of pile

A = The cross sectional area of pile

R = The pile resistance in Tonnes = 0.1 Q_u in kN

The efficiency of hammer blow (η_b) depends on the weight of hammer (W), the weight of pile, anvil and helmet follower (P) and the coefficient of resistance (e).

a). For W > e P

 $\eta_b = \frac{W+\;e^2\;P}{W+P}$

b). For W < e P

$$\eta_{b} = \frac{W + e^{2} P}{W + P} - \left(\frac{W - e P}{W + P}\right)^{2}$$

Here, the coefficient of restitution (e) varies from zero for a deteriorated condition of the head of pile to 0.5 for a steel ram of double acting hammer striking on steel anvil and driving a reinforced concrete pile.

e = 0.4, for a C.I. ram of a single acting or drop hammer striking on the head of R.C.C pile and that striking on a well conditioned driving cap and helmet with hard wood on R.C.C Pile, e = 0.25

3). Danish Formula

According to Danish formula (1929), the ultimate load capacity (Q_U) of a driven pile is given by

$$Q_{U} = \frac{W h \eta_{h}}{S + \frac{S_{0}}{2}} = \frac{E_{n} \eta_{h}}{S + \frac{S_{0}}{2}}$$

Here

$$S_0 = \left[\frac{2\eta_h(WhD)}{AE}\right]^{\frac{1}{2}} = \left[\frac{2\eta_h(E_nD)}{AE}\right]^{\frac{1}{2}}$$

 S_0 = The elastic compression of pile

W = The weight of hammer in 'kN'

h = The height of free fall of the ram (or) hammer in 'cm'

 $E_n = W.h =$ The rated energy of hammer in kN-cm

S = The final set (or) the penetration of pile per one hammer blow in 'cm' and

 $\eta_h = The \ hammer \ efficiency$

D = The length of pile

A = The cross sectional area of pile and

E = The modulus of elasticity of pile material

The allowable load (Qa) is found by taking a factor of safety of 3 to 4.

Taking $Q_U = 3 Q_a$

The final set per blow (S) is given by

$$S = \frac{W h \eta_h}{3Qa} = \frac{1}{2}S_0$$

Pile Load Test



The pile load test is the most reliable field test for determining the load carrying capacity of a pile. The pile load test set-up generally consists of two anchor piles supporting an anchor girder or a reaction at their top as shown in the figure.

The test pile is installed between the anchor piles in the same manner as the foundation piles are installed. The test pile should be at least 3B (or) 2.5 m clear distance from the anchor pile. Here 'B' is the width or diameter of the test pile.

The load is applied through the hydraulic jack resting on the top of the test pile. The measurements of pile settlements are taken with respect to three dial gauges connected to the test pile.

The test is conducted after a rest period of 3 days after the installation of test pile in sandy soils and after a period of one month in silty and clayey soils.

The load is applied on the reaction beam in equal increment of about 20% of the allowable load. The settlements of the test pile are recorded by using the three dial gauges. Each stage of the loading is maintained till the rate of movement of the pile top is not more than 0.1 mm per hour in sandy soils and 0.02 mm per hour in clayey soils.

Under each load increment the settlements are observed at 0.5, 1, 2, 4, 8, 12, 16, 20, 60 minutes. The loading on the test pile should be continued up to twice the safe load or the load at which the total settlement reaches a specified value.

The load is removed in the same decrements at 1 hour interval and the final rebound is recorded 24 hours after the entire load has been removed.



The figure shows the load – settlement curve for loading as well as for unloading obtained from the pile load test.

For any given load the net pile settlement (S_n) is given by

 $S_n = S_t \text{ - } S_e$

Here

 S_t = The total settlement (Gross settlement)

 S_e = The elastic settlement (Rebound settlement)



The figure shows the load – settlement curve obtained from the pile load test. The ultimate load (Q_U) may be determined as the abscissa of the point where the load – settlement curve changes to a steep straight line.

Alternatively, the ultimate load (Q_U) may be determined as the abscissa of the point of intersection of the initial and final tangents of the load – settlement curve.

The allowable load (Q_a) is usually taken as one - half of the ultimate load.

According to IS : 2911 - Part 4 - 1974, the allowable load (Q_a) of the pile may be taken as one of the following whoever is less

1). 50% of the load at which the total settlement is equal to one – tenth of the diameter of the pile.

2). Two – thirds of the load which causes a total settlement of 12 mm

3). Two – thirds of the load which causes a net settlement of 6 mm.

UNIT - IV

STABILITY OF EARTH SLOPES

SYLLABUS:

UNIT – I Stability of Slopes: Infinite and finite earth slopes in sand and clay – types of failures – factor of safety of infinite slopes – stability analysis by Swedish arc method, standard method of slices – Taylor's Stability Number-Stability of slopes of dams and embankments - different conditions.

INTRODUCTION

The earth slope is an unsupported, inclined surface of a soil mass. The earth slopes are of two types 1). Natural earth slopes and 2). Man made earth slopes.

The natural earth slopes are those that exist in nature and are formed by natural causes. Such slopes exist in hilly areas. The sides of cuttings, the slopes of embankments constructed for roads, railways lines, canals etc and the slopes of earth dams constructed for storing water are examples of man made slopes. The figure shows some of the examples of man made earth slopes.



The earth slopes are further classified as

1). Infinite slopes and 2). Finite slopes.

The term infinite slope is used to designate a constant slope of infinite extent. Slopes extending to infinity do not exist in nature. An infinite slope is one which represents the boundary surface of a semi-infinite soil mass inclined to the horizontal and having constant soil properties for all identical depths below the surface. However in practice if the height of the slope is very large, it may be considered as infinite slope.

A slope of limited extent, bounded by a base and a top surface is called a finite slope. The examples of finite slopes are the inclined faces of embankments, earth dams, cuttings etc.

The cost of earth work would be minimum if the earth slopes are made steepest. However very steep slopes may not be stable. The flat earth slopes may be stable. But in flat earth slope the cost of earth work would increases. Hence a compromise has to be made between the

economy and safety and the earth slopes provided are neither too steep nor too flat. In other words, the steepest earth slopes which are stable and safe should be provided.

The failure of a earth slope may lead to loss of life and property. It is, therefore, essential to check the stability of the proposed earth slopes. With the development of modern methods of testing of soils and stability analysis, a safe and economical design of a earth slope is possible. The geotechnical engineer should have a thorough knowledge of the various methods for checking the stability of slopes and their limitations.

The failure of a earth slope occurs when a large mass of soil slides along a plane or a curved surface involving a downward and outward movement of the soil mass away from the slopping surface. The surface along which the soil mass slides when the failure of an earth slope occurs is known as **critical surface of failure**.

It is evident that the failure of an earth slope occurs when the forces tending to cause the sliding or slipping are greater than those forces tending to restore or stabilise the soil mass along the critical surface of failure.

The sliding or slipping in a earth slope takes place mainly due to 1). The gravitational force 2). The force due to seepage water 3). Erosion of the surface of slope due to flowing water 4). The sudden lowering of water adjacent to a slope and 5). Forces due to earthquakes. The forces which resist the failure of earth slope are shear strength of the soil and/or frictional force, depending on the soil being cohesive or cohesionless.

The factors leading to the failure of earth slopes may be classified into two categories.

1). The factors which cause an increase in the shear stress. The shear stress in soil mass may increase due to weight of water causing saturation of soils, surcharge loads, seepage pressure or any other cause. The stresses are also increased due to steepening of slopes either by excavation or by natural erosion.

2). The factors which cause an decrease in the shear strength of the soil. The loss of shear strength of soil may occur due to an increase in the water content, increase in pore water pressure, shock or cyclic loads, weathering or any other cause.

Most of natural slope failures occur in rainy season, as the presence of water causes both increase in the shear stress and decrease in shear strength of soil.

Basic Assumptions in the Stability Analysis of Earth Slopes

The following assumptions are generally made in stability analysis of earth slopes.

1). The soil mass is assumed as homogeneous.

2). The stress system is assumed to be two-dimensional. The stresses in the third direction (perpendicular to the section of the soil mass) are taken as zero.

3). It is assumed that Coulomb's equation for shear strength is applicable and the strength parameters C and \emptyset are known.

4). It is assumed that seepage conditions and water levels are known, and the corresponding pore water pressures can be estimated.

5). The conditions of plastic failure are assumed to be satisfied along the critical surface. In other words, the shearing strains at all points of the critical surface are large enough to mobilise all the available shear strength.

Depending upon the method of analysis, some additional assumptions are made regarding the magnitude and distribution of forces along various planes.

In the analysis of stability of earth slopes, the resultant of all the actuating forces trying to cause the failure is determined. An estimate is also made on the available shear strength trying to resist the failure. The factor of safety of the earth slope is determined from the available resisting forces and the actuating forces.

DIFFERENT FACTORS OF SAFETY

In the stability analysis of earth slopes, the following factors of safety are normally used.

1). Factor of safety with respect to shear strength (\mathbf{F}_{s})

The factor of safety with respect to shear strength (F_s) is defined as the ratio of the maximum shear strength (s) to the average value of mobilized shear strength (τ_m).

Thus $F_s = \frac{s}{\tau_m}$ ----- (1)

Here $F_s =$ The factor of safety with respect to shear strength

s = The maximum shear strength and

 τ_m = The mobilised shear strength (equal to applied shear stress)

In terms of cohesion intercept (C) and the angle of shear resistance (${\it \emptyset}$)

The maximum shear strength = $s = C + \overline{\sigma} \tan \phi$ and

The mobilised shear strength = $\tau_m = C_m + \overline{\sigma} \tan \phi_m$

Here C_m = The mobilised cohesion and

$\overline{\sigma}$ = The effective pressure

 $\therefore \qquad F_{\rm s} = \frac{C + \overline{\sigma} \tan \emptyset}{Cm + \overline{\sigma} \tan \phi_m} \quad ----- (2)$

2). Factor of safety with respect to cohesion (Fc)

The factor of safety with respect to cohesion (F_c) is defined as the ratio of the available cohesion intercept (C) to the mobilised cohesion intercept (C_m).

Thus
$$F_c = \frac{c}{c_m}$$
 ----- (3)

Here F_c = The factor of safety with respect to cohesion

C = The available cohesion intercept and

 C_m = The mobilised cohesion intercept

3). Factor of safety with respect to friction (F_{\emptyset})

The factor of safety with respect to friction (F_{\emptyset}) is defined as the ratio of the available frictional strength ($\overline{\sigma} \tan \emptyset$) to the mobilised frictional strength ($\overline{\sigma} \tan \emptyset_m$).

Thus $\mathbf{F}_{\emptyset} = \frac{\overline{\sigma} \tan \emptyset}{\overline{\sigma} \tan \emptyset_m}$ ------ (4)

Here F_{\emptyset} = The factor of safety with respect to friction

 \emptyset = The angle of shearing resistance and

For small angles of shearing resistance the equation (4) can be expressed as

$$\boldsymbol{F}_{\boldsymbol{\emptyset}} = \frac{\boldsymbol{\emptyset}}{\boldsymbol{\emptyset}_m} \quad \dots \quad (5)$$

From equation (2) we have $F_s = \frac{C + \overline{\sigma} \tan \emptyset}{Cm + \overline{\sigma} \tan \emptyset_m}$

Rearranging the above equation we get

$$C_{\rm m} + \overline{\sigma} \tan \phi_{\rm m} = \frac{C + \overline{\sigma} \tan \phi}{F_{\rm s}}$$

From the above equation (6) we get

$$C_m = \frac{C}{F_s} \rightarrow F_s = \frac{C}{C_m}$$

But $\frac{c}{c_m}$ = The factor of safety with respect to cohesion (F_c)

$$\therefore \mathbf{F}_{s} = \mathbf{F}_{c} \quad ----- \quad (7)$$

Similarly from equation (6) we get

 $\frac{\overline{\sigma} \tan \phi}{F_{s}} = \overline{\sigma} \tan \phi_{m}$ $\therefore F_{s} = \frac{\overline{\sigma} \tan \phi}{\overline{\sigma} \tan \phi_{m}}$

But $\frac{\overline{\sigma} \tan \phi}{\overline{\sigma} \tan \phi_m}$ = The factor of safety with respect to friction = F_{ϕ}

$$\therefore \mathbf{F}_{\mathrm{s}} = F_{\emptyset} - \dots - (8)$$

From equations (7) and (8) we get

 $F_s = F_c = F_{\emptyset}$

Therefore in the stability analysis of earth slopes, generally the three factors of safety are taken as equal.

TYPES OF EARTH SLOPE FAILURES

The earth slope may have any one of the following failures.

1). Rotational failure.

This type of failure occurs by rotation along a slip surface by downward and outward movement of the soil mass as shown in figure. The slip failure is generally circular for homogeneous soil conditions and non-circular in case of non-homogeneous soil conditions. Rotational slip failures are further sub-divided into the following three types.

a). Toe failure, in which the failure occurs along the surface that passes through the toe as shown in figure

b). Slope failure, in which the failure occurs along the surface that intersects the slope above the toe as sown in figure.

c). Base failure, in which the failure surface passes below the toe as shown in figure.

The slope failure occurs when a weak plane exists above the toe. The base failure occurs when a weak stratum lies beneath the toe. If a strong stratum exists below the toe, the slip surface of the base failure is tangential to that stratum. In all other cases, the failures are toe failures. The toe failures are most common earth slope failures.



2). Translational failure. A constant earth slope of unlimited extent and having uniform soil properties at the same depth below the free surface is known as an infinite slope.

In practice, the earth slopes which are of considerable extent and in which the conditions on all verticals are adequately represented by average conditions are designated as infinite slopes.

Translational failure occurs in an infinite slope along a long failure surface parallel to the slope as shown in figure. The shape of the failure surface is influenced by the presence of any hard stratum at a shallow depth below the slope surface. Translational failures may also occur along the slopes of layered materials.



3). Compound failure. The compound failure is a combination of the rotational slips and translational slips as shown in figure. A compound failure surface is curved at the two ends and plane in the middle portion. A compound failure generally occurs when a hard stratum exists at considerable depth below the toe.



4). Wedge failure. A failure along an inclined plane is known as plane failure or wedge failure or block failure. It occurs when distinct blocks and wedges of the soil mass become separated.

A plane failure is similar to translational failure in many respects. However, unlike translational failure which occurs in an infinite slope, a plane failure may occur even in a finite slope consisting of two different materials or in a homogeneous slope having cracks, fissures, joints or any other specific planes of weakness.



5). Miscellaneous failures. In addition to above four types of failures, some complex types of failures in the form of spreads and flows may also occur.
The Wedge Failure Stability Analysis of Finite Soil Slope

The failure along an inclined plane surface is known as wedge failure. The wedge failure is similar to translational failure. However, the translational failure occurs in an infinite soil slope and the wedge failure occurs in finite soil slope consisting of two different materials or in a homogeneous slope having cracks, fissures, joints or any other specific planes of weakness.

The wedge failure occurs when a soil slope has a specific plane of weakness. The figure shows a finite soil slope 'AB" subjected to wedge failure along the Plane failure surface "AD'. In wedge failure there is a tendency of the upper mass to slide downward along the failure plane of contact 'AD'.



Let,

i = The angle made by the finite soil slope 'AB" with horizontal

 α = The angle made the wedge failure plane 'AD' with horizontal and

W = The weight of soil wedge 'ABD' per unit thickness

Here

The weight of soil wedge 'ABD' = $W = \gamma$ x Volume of Soil Wedge 'ABD'

 γ = The bulk unit weight of soil

Along the plane AD, the weight of the soil wedge (W) can be resolved into two components

1). The normal component (N) and

2). The tangential component (T).

Here a). The normal component = $N = W \cos \alpha$ ------ (1)

b). The tangential component = $T = W \sin \alpha$ ------ (2)

The tangential component 'T' is the force causing the wedge failure of soil slope.

The maximum shearing force (S) tending to resist the wedge failure of soil slope is given by (S)

 $S = C + N \tan \emptyset = C + (W \cos \alpha) \tan \emptyset$ ------(3)

Let

c = Cohesion per unit length of soil along the failure plane 'AD' and

L = Length of failure plane 'AD'

Here

1). Total cohesion of soil = C = c L and

2).S = cL + (W cos α) tan \emptyset ------ (4)

Now, the factor of safety of shear strength against shear failure (Fs) given by is given by

 $F_{s} = \frac{\text{Maximum Shear force resisting the failure}}{\text{Shear force causing the failure}} = = \frac{S}{T} = \frac{\text{cL} + (W \cos \alpha) \tan \phi}{W \sin \alpha} - \dots - (4)$

The Culmann's Method of Stability Analysis of Finite Soil Slope

The Culmann's method is used for the approximate stability analysis of homogeneous finite soil slopes. In this method a plain failure surface passing through the toe is assumed. A plain failure surface is not a correct assumption for a homogeneous soil. However, it is a simple failure mechanism and is described for purpose of illustration and for determination of the approximate value of the factor of safety.



Let us consider the equilibrium of the triangular soil wedge 'ABD' formed by the inclined soil slop surface 'AB' and the assumed plane failure surface 'AD' as shown in figure.

Let i = The angle made by the inclined soil surface 'AB' with horizontal and

 α = The angle made by the plane failure surface 'AD' with horizontal

 C_m = The total mobilised cohesive force acting on the failure surface 'AD'

L = The length of plane failure surface 'AD' and

H = The vertical height of inclined soil surface 'AB'

From the diagram

$$\cos (90^{0} - i) = \frac{H}{AB} \qquad \qquad \text{Sin } i = \frac{H}{AB}$$
$$\therefore AB = \frac{H}{\text{Sin } i}$$

Stability Analysis of Finite Slopes

The finite slope is bounded by a base and a top surface. The unlined inclined faces of earth dams. embankments, excavations etc., are all finite slopes. Thus, the stability analysis of finite slope is of vital importance for civil engineers.

The failure of finite slopes occurs along a curved surface. In stability analysis of finite slopes the curved failure surface is usually replaced by an arc of a circle or a logarithmic spiral.

Two basic types of failure of a finite slope may occur : 1). Slope failure 2). Base failure

1). Slope failure

If the failure occurs along a surface of sliding or a surface of slippage that intersects the slope at or above the toe is known as slope failure.

The slope failure is called a **face failure** if the failure surface passes above the toe.

The slope failure is called a **toe failure** if the failure surface passes through the toe.

2). Base failure

If the soil beneath the toe of the slope is weak, the failure occurs along a surface that passes at some distance below the toe of the slope. Such a type of failure is called base failure.

The slope failure and the base failure of finite slopes are as shown in figure.

The ratio of the total depth '(H+D)' to the depth 'H' is known as 'Depth factor (D_f) '.

The depth factor = $D_f = \frac{H+D}{H}$

For toe failure $D_f = 1$ and for base failure $D_f > 1$.

a). Face Failure



Slope Failure

b). Toe Failure



The stability analysis of finite slopes involves the following steps

1). Assuming a possible slip surface or failure surface.

2). Studying the equilibrium of the forces acting on this surface and determine the factor of safety

3). Repeating the procedure until the worst slip surface, for which minimum factor of safety, is found.

The stability of finite slopes can be analysed by a number of methods. The following methods are commonly adopted.

1). The Friction circle method.

2). The Taylor's method..

3). The Swedish circle method.

4). The Bishop's method.

The $\emptyset_u = 0$ analysis

In case of fully saturated clays under undrained conditions, the stability of slopes can be checked in terms of total stresses. Such a condition can occurs in soil slopes immediately after construction. In this case $\phi_u = 0$ and $c = c_u$.

The failure surface is assumed to be a circular arc 'AB'. The figure shows a failure surface with centre 'O' and radius 'r'. The weight 'W' above the failure surface causes the instability. For equilibrium, the shear stress to be mobilised along the failure surface can be expressed as

Here τ_m = The mobilised shear stress along the failure surface 'AB'

s = The shear strength of soil = c_u and

F = The factor of safety.

Now, by taking the moments about the centre 'O', we get

W x d = τ_m x L_a x r ----- (2)

Here $L_a =$ The length of circular arc 'AB' = $\frac{2\pi r\theta}{360^0}$ and

d = The lever arm of weight 'W' about the centre 'O'

Substituting equation (1) in equation (2), we get

W x d =
$$\frac{c_u}{F}$$
 x L_a x r
 \therefore F = $\frac{c_u L_a r}{W x d}$ ------(3)

If a tension crack develops and water enters the crack, the hydrostatic force ' P_w ' acts on the portion 'BC' of the arc at a height of 'h/3' from 'C'. Here 'h' is the depth of the tension crack.

The depth of tension crack is given by $h = 2 \frac{c}{v}$

The effect of tension crack is to shorten the length of failure surface along which the shearing resistance gets mobilised to 'AC'.

In this case the arc length 'L_a' is equal to 'AC'.

UNIT – V

EARTH RETAINING STRUCTURES

Introduction

We know that a soil mass is stable when the slope of the surface of the soil mass is flatter than the safe slope. At some locations where the space is limited, it is not possible to provide a flat slope and the soil is to be retained at slope steeper than the safe slope.

In such cases, a retaining structure is required to provide the lateral support to the soil mass. Generally the soil mass behind the retaining structure is either vertical or nearly vertical.

Sometimes the retaining wall maintains the soil at different elevations on its either side. In the absence of a retaining wall, the soil on the higher side would a tendency to slide and may not remain stable.

The design of the retaining structure requires the determination of the magnitude and line of action of the lateral earth pressure.

The magnitude of the lateral earth pressure depends upon a number of factors, such as the mode of the movement of the retaining wall, the flexibility of retaining wall, the properties of the soil and the drainage conditions.

As the lateral earth pressure depends upon the flexibility of the retaining wall, it is a soilstructure interaction problem.

For convenience, the retaining wall is assumed to be rigid and the soil-structure interaction effect is neglected.

The lateral earth pressure is usually computed by using the classical theories proposed by Coulomb (1773) and Rankine (1857). The general wedge theory proposed by Terzaghi (1941) is more general and is an important over the earlier theories. However, this theory is quite completed.

Different types of lateral earth pressures

The soil retained by the retaining wall is also known as the backfill.

Depending upon the movement of the retaining wall with respect to the soil retained (or) backfill, the lateral earth pressures of soil mass can classified into three categories.

- 1). At-rest earth pressure
- 2). Active earth pressure and
- 3). Passive earth pressure

1). At-rest earth pressure

The lateral earth pressure is called at-rest earth pressure, when the retained soil (or) backfill is not subjected to any lateral movement or lateral yielding.

This case occurs when the retaining wall is firmly fixed at its top and is not allowed to rotate or move laterally.

The figure shows the basement retaining walls which are restrained against the movement by the basement slab provided at their top.

In this case the soil retained by the retaining wall is not subject to any lateral movement. Hence retained soil exerts the at-rest earth pressure on the retaining walls.

The at-rest earth pressure condition is also known as the 'Elastic Equilibrium' as no part of soil mass has failed and attained the condition of plastic equilibrium.



2). Active earth pressure

The state of 'Active earth pressure' occurs when the movement of retaining wall is such that the retained soil (or) the back tends to stretch horizontally.

The retaining wall when moves away for the backfill, there is a stretching of the retained soil mass and the active state of earth pressure occurs.

The active state of earth pressure is a state of plastic equilibrium as the entire soil mass is on the verge of failure.

As shown in figure the active earth pressure develops on the retained soil when the retaining wall moves towards left.

3). Passive earth pressure

The state of 'Passive earth pressure' occurs when the movement of retaining wall is such that the retained soil (or) the back tends to compress horizontally.

The retaining wall when moves towards the backfill, there is a compression of the retained soil mass and the passive state of earth pressure occurs.

The passive state of earth pressure is a state of plastic equilibrium as the entire soil mass is on the verge of failure.

As shown in figure the passive earth pressure develops on the retained soil when the retaining wall moves towards right.

Variation of earth pressure

The figure shows the variation of earth pressure with the movement of retaining wall.

The point 'B' represents the case when there is no movement of the retaining wall. It indicates the state of 'At-rest earth pressure'.



The point 'A' in the figure indicates the case when the retaining wall moves away from the backfill. It indicates the state of 'Active earth pressure'.

As shown in figure, when the retaining wall moves away from the backfill, some portion of backfill located immediately behind the wall tries to break away from the rest of soil mass.

This wedge-shaped portion is known as the failure wedge, moves downwards and outwards.

In active earth pressure, the lateral earth pressure exerted on the retaining wall is minimum. The soil is at the verge of failure due to decrease in the lateral stress.

The horizontal strain required to reach the active state of plastic equilibrium is very small. Lambe and Whitman (1969) have shown that in dense sand, the horizontal strain required is about 0.5%. For example, for a wall of 5 m height, a movement of 0.0144 m would develop active earth pressure.

The point 'C' in the figure indicates the case when the retaining wall moves towards the backfill. It indicates the state of 'Passive earth pressure'.

As shown in figure when the retaining wall moves towards the backfill, some portion of backfill located immediately behind the wall tries to break from the rest of soil mass.

This wedge-shaped portion is known as the failure wedge, moves upwards and inwards. In passive earth pressure, the lateral earth pressure exerted on the retaining wall is maximum. The soil is at the verge of failure due to increase in the lateral stress.

Lambe and Whitman (1969) found that very little horizontal strain (about 0.5%) is required to reach one-half the maximum passive earth pressure in dense sand, but much more horizontal strain (about 5% in dense sand and 15% in loose sand) is required to reach the full maximum passive earth pressure.

Thus it may be summarised that the state of shear failure corresponding to the minimum earth pressure is the active state and that corresponding to the maximum earth pressure is the passive state.

The active state and the passive state are the two extreme conditions of plastic equilibrium. For all intermediate states, the soil is not in plastic equilibrium and it is said to be in elastic equilibrium.

The at-rest condition is a special case of an elastic equilibrium when the state of stress corresponding to the condition where there is no movement.

Earth pressure at rest

The earth pressure at rest, exerted on the back of a rigid, unyielding retaining structure, can be calculated by using the theory of elasticity, assuming the soil to be semi-infinite, homogeneous, elastic and isotropic.

Consider an element of soil at a depth 'z', being acted upon by the vertical stress ' σ_v ' and the horizontal stress ' σ_h ' as shown in figure.

There will be no shear stress and hence both ' σ_v ' and ' σ_h ' are the principal stresses.

Let 'E' and 'µ'be the modulus of elasticity and Poisson's ratio of the soil respectively.

Now, the lateral strain ' \mathcal{E}_h ' of the soil element in the horizontal direction is given by



But in 'At-Rest Earth pressure 'condition, the retained soil or backfill is not subjected to any lateral movement and hence the lateral strain ' \mathcal{E}_h ' of the soil element in the horizontal direction is equal to zero.

Hence, from equation (1) we get

$$0 = \frac{1}{E} [\sigma_h - \mu(\sigma_v + \sigma_h)]$$

$$\sigma_h = \mu(\sigma_v + \sigma_h)$$

$$\therefore \sigma_h (1 - \mu) = \mu \sigma_v$$

$$\therefore \frac{\sigma_h}{\sigma_v} = \frac{\mu}{1 - \mu}$$

But in 'At-Rest Earth pressure 'condition,

The ratio of the horizontal stress(or) the lateral stress (σ_h) to the vertical stress (σ_v) is known as the 'coefficient of earth pressure at-rest', which is represented as 'K₀'.

Now, from equation (2), the lateral stress or the lateral earth pressure ' σ_h ' is given by

$$\sigma_{\rm h} = \mathrm{K}_0 \sigma_{\rm v} - \dots - (3)$$

The lateral earth pressure at rest ' σ_h ' is also designated as ' p_0 '

:
$$p_0 = K_0 \sigma_v$$
 ----- (4)

Now, if the retained soil or the back fill is in dry condition

Then, the vertical stress ' σ_v ' at depth 'Z' from the top is given by

 $\sigma_v = \gamma Z \dots (5)$

Where γ = The unit weight of soil

Now,

From equation (4), the lateral earth pressure at rest (p_0) is given by

 $p_0 = K_0 \gamma Z$ ----- (6)

It is clear from equation (6) that the distribution of the lateral earth pressure at rest (p_0) with depth (Z) is linear.

Thus for a retaining wall of height 'H', the lateral earth pressure at rest (p_0) distribution diagram is a triangular with zero intensity at the top of retaining wall, where Z =0 and an intensity of 'K₀ γ H' at the base of the retaining wall, where Z = H.

The figure shows the lateral earth pressure (p_0) distribution with depth(Z), when the soil is dry.



The earth pressure (p_0) at the bottom of retaining wall at depth 'H' is given by

$$p_0 = K_0 \gamma H$$
 -----(7)

Now, the total earth pressure force (P₀) at rest per unit length of the retaining wall is given by

$$P_0 = \int_0^H K_0 \gamma Z \, dz$$

$$\therefore P_0 = \frac{1}{2} K_0 \gamma H^2 - \dots (8)$$

The total earth pressure force (P₀) at rest will act at $\frac{1}{3}$ H from the base of the wall.

| S.No | Soil type | K_0 |
|------|--------------------------|-------|
| 1 | Loose sand | 0.4 |
| 2 | Dense sand | 0.6 |
| 3 | Sand compacted in layers | 0.8 |
| 4 | Soft clay | 0.6 |
| 5 | Hard clay | 0.5 |

Let us consider the case of a retaining wall with ground water table at the ground surface as shown in figure.



In this case

a). The total vertical stress ' σ_v ' at point 'A' at a depth 'Z' is given by

$$\sigma_{\rm v} = \gamma_{\rm sat} \, Z$$

b). The pore water pressure (u) at point 'A' at a depth 'Z' is given by

$$u=\gamma_w Z$$

c). The effective vertical stress ' $\overline{\sigma}_v$ ' at point 'A' at a depth 'Z' is given by

$$\overline{\sigma}_{v} = \sigma_{v} - u = \gamma_{sat} Z - \gamma_{w} Z = \gamma' Z$$
(9)

Now

From equation (2), the effective lateral earth pressure at rest ' \bar{p}_0 ' at a depth 'Z' is given by

$$\overline{p}_0 = K_0 \overline{\sigma}_v = K_0 \gamma' Z$$
------(10)

Now, the total lateral earth pressure (p_h) at restat a depth 'Z' is equal to the sum of the effective lateral earth pressure at rest (\bar{p}_0) and the pore water pressure (u).

Thus, it is clear from equation (11) that for a retaining wall of height 'H', the total lateral earth pressure at rest (p_h) distribution diagram is a triangular with zero intensity at the top of retaining wall, where Z =0 and an intensity of 'K₀ γ' H + γ_w H' at the base of the retaining wall where Z = H.

The figure shows the total lateral earth pressure (p_h) distribution with depth(Z), when the water table is at the top of ground surface.

Now, the total earth pressure force (P_h) at rest per unit length of the retaining wall is given by

$$P_{h} = \frac{1}{2} H[K_{0}\gamma'H + \gamma_{w}H] - \dots (12)$$

Let us consider the case of a retaining wall with ground water table at a depth 'd' below the ground surface as shown in figure.



The vertical stress ' σ_v ' at depth 'd' above the ground water table from the top is given by

$$\sigma_v = \gamma d$$

Where γ = The unit weight of soil above the ground water table

RANKINE'S EARTH PRESSURE THEORY

Rankine(1857) developed his theory of lateral earth pressure when the backfill consists of dry and cohesionless soil. The Rankine's theory was later extended by Resal (1910) and Bell (1915) to be applicable to cohesive soils.

In his earth pressure theory, Rankine (1857) considered the equilibrium of a soil element within a soil mass bounded by a plane surface.

The following assumptions were made in Rankine's earth pressure

1). The soil mass is homogeneous and semi-infinite.

2). The soil is dry and cohesionless.

3). The ground surface is a plane, which may be horizontal or inclined.

4). The back of the retaining wall in contact with the backfill is smooth and vertical.

5). The soil element is in a state of plastic equilibrium, i.e., at the verge of failure.

In Rankine's earth pressure theory, the expressions for active earth pressure and passive earth pressure are developed as explained below.

(1). Active Earth Pressure of Dry (or) Moist Cohesionless soil Backfill

Consider a retaining wall of height 'H' with a vertical back, retaining a dry or moist mass of cohesionless soil, the surface of which is level with the top of the retaining wall as shown in figure.

Consider an element of dry soil at a depth 'z', being acted upon by the vertical stress ' σ_v ' and the horizontal stress ' σ_h ' as shown in figure. There will be no shear stress and hence both ' σ_v ' and ' σ_h ' are the principal stresses.



Initially the soil element is at rest, hence the horizontal stress ' σ_h ' is given by

$$\sigma_{\rm h} = K_0 \sigma_{\rm v}$$

Where $\sigma_v =$ The vertical stress at the soil element $= \gamma Z$

Now, at the soil element the stress ' σ_h ' and ' σ_v ' are , respectively, the minor and the major principal stresses and are indicated by the points 'A' and 'B' in the Mohr circle of stresses as shown in figure. Under these stress conditions in this state of rest, the soil element is in a state of elastic equilibrium.

However, the horizontal movement (or) the deformation of the soil mass can change this situation.

For example, if the soil mass gets stretched horizontally, the vertical principal stress (σ_v) remains constant but the horizontal principal stress (σ_h) is decreased.

Due to this the point 'A' in the Mohr circle shifts to the position 'A' and the diameter of the Mohr circle increases.

When the horizontal principal stress (σ_h) reaches a limiting minimum value condition, the point 'A' in the Mohr circle shifts to the position 'A'' and the Mohr circle touches the failure envelope as shown in figure.



Thus, at this limiting minimum condition of horizontal principal stress, the soil is at the verge of shear failure and it has said to attained the Rankine's active state of plastic equilibrium.

The horizontal principal stress (σ_h) at that state is also known as the lateral active earth pressure (p_a) .

The figure shows the Mohr circle when active conditions are developed. The point 'E' represents the active earth pressure ' p_a '.



Now from the above Mohr circle, we get

The lateral active earth pressure $= p_a = OE = OC - CE$

But $CE = CD = OC \sin \emptyset$

: $p_a = 0C - 0C \sin \emptyset = = OC (1 - \sin \emptyset) - (1)$

Now, t

The vertical stress = $\sigma_v = OB = OC + CB$

But $CB = CD = OC \sin \emptyset$

 $\therefore \ \sigma_{v} = OC + OC \sin \emptyset = = OC (1 + \sin \emptyset) - \dots (2)$

Now, from equations (1) and (2) we get

$$\therefore \frac{p_a}{\sigma_v} = \frac{(1 - \sin \emptyset)}{(1 + \sin \emptyset)}$$