MODERN ENGINEERING
&
MANAGEMENT STUDIES

SUBJECT NAME – MATHEMATICS – II

SUBJECT CODE – 23BS1004

LECTURE NOTES

B.TECH 1st YEAR – SEM- II (2024-2025)

DEPARTMENT OF BASIC SCIENCE
AND
HUMANITIES

Module-I

Date - 18th NOV

Differential Eqⁿ

Basic term in D.E :- let $y = f(x)$ be a function then $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots\ldots \frac{d^ny}{dx^n}$ all are derivatives of y.

→ An eqⁿ which contains derivatives of 1 or more dependent variables with respected to one or more independent variable is called differential Eqⁿ.

Ordinary Differential eqⁿ :-

→ An eqⁿ which contains ordinary derivatives of One or more depent variable with respect to one independent variable

Dependent and Independent Variable in DE

gi. $\frac{dy}{dx}$ is derivatives in DE, then y is called dependent variable and 'x' is independent variable.

$x^2 + y^2 = a^2$ D.E. w.r.t x

$2x + 2y \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} + \frac{x}{y} = 0$

dy/dx = ODE

→ eg:- $y^2 = 4ax$

$\Rightarrow \frac{d}{dx}(y^2) = \frac{d}{dx}(4ax)$

$\Rightarrow 2y\frac{dy}{dx} = 4a$

$\Rightarrow 2y\frac{dy}{dx} = 4a \Rightarrow 2y\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{\cancel{4}^{2}a}{2y} \qquad \Rightarrow 2y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

$\left[\begin{array}{l}\text{Order} = 2 \\ \text{Degree} = 1\end{array}\right.$

$\Rightarrow \boxed{\frac{dy}{dx} - \frac{2a}{y} = 0}$ ——— (1)

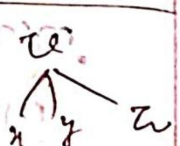
Order = 1 Degree = 1

$\Rightarrow \frac{d^2y}{dx^2} - 2a \cdot \frac{d(1/y)}{dx} = 0$

$\Rightarrow \frac{d^2y}{dx^2} - 2a\left(y\frac{d1}{dx} - 1\frac{dy}{dx}\right) = 0$

$\Rightarrow \frac{d^2y}{dx^2} - 2a\left(0 - \frac{dy}{dx}\right) = 0$

$\Rightarrow \frac{d^2y}{dx^2} + \frac{2a}{y^2}\left(\frac{dy}{dx}\right) = 0$



$\frac{\partial u}{\partial x}$ PDE

q: 27

Degree of a DE:- Highest derivative in a
Degree highest derivative in a D.E:- differential eqn.

order:- Highest value in a D.E.

$$\frac{d^2y}{dx^3} + 3\left(\frac{d^3y}{dx^3}\right)^4 + 5x^2\frac{d^3y}{dx^3} = 0$$

(1) $\frac{d^4y}{dx^4} + \sin\left(\frac{d^2y}{dx^2}\right) = 0$ $\left[\begin{matrix} O-4 \\ d=\infty \end{matrix}\right]$ $O-4$ $D-\infty$

(2) $\left(\frac{ds}{dt}\right)^4 + 3\left(\frac{d^2s}{dt^2}\right) = 0$ $\left[\begin{matrix} O=2 \\ d=1 \end{matrix}\right]$ $O-2$ $D=1$

(3) $\frac{d^2y}{dx^2} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$ $\left[\begin{matrix} O=2 \\ d=2 \end{matrix}\right]$ $O-2$, $D\ 2$

$D=\infty$ is not defined, if D.E envolves $e^{\left(\frac{dy}{dx}\right)}$, $\log\left(\frac{dy}{dx}\right)$, $\tan\left(\frac{dy}{dx}\right)$, $\sin\frac{dy}{dx}$, $\cos\frac{dy}{dx}$, $\tan^{-1}\left(\frac{dy}{dx}\right)$ is not defined the degree of a D.E but order is defined as usual.

Variable separable method:-

D.E of the form

$N(y)dy = m(x)dx$ ——— (1)

both side integrating

$\int N(y)dy = \int m(x)dx$ ——— (2)

$m(x)\,P(y)\,dx = N(y)\,Q(x)\,dy$

$\frac{m(x)}{Q(x)}dx = \frac{N(y)}{P(y)}dy$

* $(x^2+1)dy + (y^2+1)dx = 0$

$\Rightarrow (x^2+1)dy = -(y^2+1)dx$

$\Rightarrow \int\frac{dy}{(y^2+1)} = \int\frac{dx}{-(x^2+1)}$

$\Rightarrow \int\frac{dy}{y^2+1} = \int\frac{dx}{-(x^2+1)}$

$\Rightarrow \tan^{-1}y = -\tan^{-1}x + c$

$\Rightarrow \tan^{-1}x + \tan^{-1}y = c$

HW

① $x(1+y^2)dx - y(1+x^2)dy = 0$

a) $x(1+y^2)dx = y(1+x^2)dy$

$\Rightarrow \frac{x}{(1+x)^2}dx = \frac{y}{(1+y)^2}dy$

Dibbrecant both side

$\int \frac{x}{1+x^2}dx = \int \frac{y}{1+y^2}dy$

$\Rightarrow 1+x^2 = t$ $\quad$ $\Rightarrow 1+y^2 = u$

$2x\,dx = dt$ $\quad$ $2y\,dy = du$

$x\,dx = \frac{1}{2}dt$ $\quad$ $y\,dy = \frac{1}{2}du$

$\Rightarrow \frac{1}{2}\int \frac{1}{t}dt = \frac{1}{2}\int \frac{1}{u}du$

$\Rightarrow \ln t = \ln u + \ln c$

$\Rightarrow \ln(1+x^2) - \ln(1+y^2) = \ln c$

$\Rightarrow \ln\left(\frac{1+x^2}{1+y^2}\right) = \ln c$

$\Rightarrow \frac{1+x^2}{1+y^2} = c$

② $x\frac{dy}{dx} = \sqrt{1-y^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{x}$

$\Rightarrow x\,dy = dx\sqrt{1-y^2}$

$\Rightarrow \int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{x}$ $\Rightarrow \sin^{-1}y - \log x = c$

$\Rightarrow \sin^{-1}y = \log|x| + c$

③ $\frac{dx}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} = 0$

$= \int \frac{dx}{\sqrt{1-x^2}} = \int -\frac{dy}{\sqrt{1-y^2}}$

$\Rightarrow \sin^{-1}x = -\sin^{-1}y + c$

$\Rightarrow \sin^{-1}x + \sin^{-1}y = c$

Q. $\sec^2x\tan y\,dx + \sec^2y\tan x\,dy = 0$

$\Rightarrow \sec^2x\tan y\,dx = -\sec^2y\tan x\,dy$

$\Rightarrow \int \frac{\sec^2x\,dx}{\tan x} = -\int \frac{\sec^2 x}{\tan y}dy$

$\Rightarrow \int \frac{dt}{t} = \int \frac{dz}{z}$

$\Rightarrow \ln t = -\ln z + \ln c$

Date - 19/11/22

① Check if the differential eqn:-

$y' = xy - 21 + 3y - 7x$ by using V.S.M

$\Rightarrow y' = xy - 7x + 3y - 21$

$\frac{dy}{dx} = xy - 7x + 3y - 21$

$\frac{dy}{dx} = x(y-7) + 3(y-7)$

$\frac{dy}{dx} = (y-7)(x+3)$

$\Rightarrow \frac{dy}{(y-7)} = x+3(dx)$

$\Rightarrow \int \frac{dy}{y-7} = \int x+3\,dx$

$\Rightarrow \ln(y-7) = \frac{x^2}{2} + 3x$

(2) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$. $[r(1) = 2]$

$\Rightarrow \int r^2 dr = \int \frac{d\theta}{\theta}$

$\Rightarrow \frac{r^{-1}}{-1} = \ln\theta + c$

$\Rightarrow -\frac{1}{r} = \ln\theta = 1/2$

$\Rightarrow \frac{2^{-1}}{-1} = \ln\theta + c$

$\Rightarrow -\frac{1}{2} = 0 + c$

$\Rightarrow c = -1/2 \Rightarrow -\frac{1}{r} = \ln\theta \cdot 1/2$

(3) $\frac{dy}{dx} = 6y^2x$ $\quad [y(1) = 1/25]$

$\int \frac{dy}{y^2} = \int 6x\,dx$

$\Rightarrow -\frac{1}{y} = 6x \cdot \frac{x^2}{2} + c$

$\Rightarrow -\frac{1}{y} = 3x^2 + c$

$\Rightarrow -\frac{1}{y} = 3x^2 - 28$

$\Rightarrow \frac{1}{y} = 28 - 3x^2$

(4) $y' = \frac{xy^3}{\sqrt{1+x^2}}$ $\qquad y(0) = 1$

$\Rightarrow \frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

$\Rightarrow \int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}} dx$

Let $t = 1+x^2$

$\Rightarrow \frac{dt}{dx} = 2x$

$\Rightarrow dt = (2x) dx$

$\Rightarrow \frac{dt}{2} = x dx$

$\Rightarrow y^{-3} dy = \frac{1}{2} \int \frac{dt}{t}$

$\Rightarrow \frac{y^{-2}}{-2} = \frac{1}{2} \times \frac{t^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}$

$\Rightarrow \frac{y^{-2}}{-2} = \frac{1}{2} \times \frac{t^{1/2}}{1/2} + c$

$\Rightarrow \frac{-1}{2y^2} = \sqrt{t} + c$

$\Rightarrow -\frac{1}{2y^2} = \sqrt{1+x^2} + c$

$\Rightarrow \frac{-1}{2(1)^2} = -\sqrt{1} + c$

$\Rightarrow \frac{-1}{2} = 1 + c$

$\Rightarrow c = -\frac{1}{2} + 1$

$\Rightarrow c = -3/2$

HW

1. $\frac{dy}{dx} = \frac{x^3+3}{y^2+1}$

$\Rightarrow \cancel{\int \frac{dy}{y^2+1}} = \cancel{\int \frac{dx}{x^3+3}}$

$\Rightarrow (y^2+1) dy = x^3 + 3 dx$

$\Rightarrow \int y^2 dy + dy = \int x^3 dx + 3 dx$

$\Rightarrow \frac{y^3}{3} dy + y = \frac{x^4}{4} + 3x + 4$

Date-23 NOV

Date - 23 NOV

Homogeneous equation:-

→ A function f(x,y) is called homogenous of degree n.

If $F(tx, ty) = t^n f(x,y)$

Ex:- $x^2+y^2 = t^n(x,y)$

$x = xt$

$y = yt$

$x^2t^2+y^2t^2$

$= t^2(x^2+y^2)$

$= t^n f(x,y)$

Ex = $x\cot^{-1}\left(\frac{y}{x}\right)$ - is a homogeneous eqn.

Def:- The differential equation

$\underline{M(x,y)}\,dx + \underline{N(x,y)}\,dy = 0$

is said to be homogeneous if M(x,y) and N(x,y) are homogeneous function of the same degrees.

for eg:- $(x^2 + xy)\,dx + y^2 dy = 0$

Q1. Solve $x^2y\,dx - (x^3+y^3)\,dy = 0.$

Soln- The eqn can be written as

$$\frac{dy}{dx} = \frac{x^2y}{x^3+y^3}$$

$x = xt$

$y = yt$

$= \frac{x^2t^2ty}{x^3t^3+y^3t^3}$

$= \frac{x^2y\cdot t^3}{t^3(x^3+y^3)}$

$= \frac{x^2y}{x^3+y^3}$

Putting $y = vx$

⇒ $\cancel{\frac{dy}{dx}}$

Q. $x^2y\,dx - (x^3+y^3)\,dy = 0$

$\Rightarrow (x^3+y^3)\,dy = x^2y\,dx$

$\Rightarrow \frac{dy}{dx}$

$y = vx$

$\Rightarrow \frac{dy}{dx} = v\frac{dx}{dx} + x\frac{dv}{dx}$

$= v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2vx}{x^3+v^3x^3}$

$= \frac{x^3v}{x^3(1+v^3)}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{v}{(1+v^3)}$

$x\frac{dv}{dx} = \frac{v}{(1+v^3)} - v$

$= \frac{v - v(1+v^3)}{1+v^3}$

$= \frac{v - v - v^4}{(1+v^3)}$

$x\frac{dv}{dx} = -\frac{v^4}{(1+v^3)}$

By VSM $= \frac{1+v^3}{v^4}dv = -\frac{dx}{x}$

Integration both side

$\int \frac{1+v^3}{v^4}dv = -\int \frac{dx}{x}$

$\Rightarrow \int \frac{1}{v^4}dv + \int \frac{v^3}{v^4}dv = -\ln x + \ln c$

$\Rightarrow \int v^{-4}dv + \int \frac{1}{v}dv = -\ln x + \ln c$

$\Rightarrow -\frac{1}{3v^3} + \ln v = -\ln x + \ln c$

$\Rightarrow \ln v + \ln x - \ln c = \frac{1}{3v^3}$

$\Rightarrow \ln vx - \ln x = \frac{1}{3v^3}$

$\Rightarrow \ln \frac{vx}{c} = \frac{1}{3v^3}$

[Let $y = vx$
$v = y/x$]

$\Rightarrow \frac{vx}{c} = e^{\frac{1}{3}v^3}$

$\Rightarrow \frac{\frac{y}{x} \cdot x}{c} = \frac{1}{3\left(\frac{y}{x}\right)^3}$

$\Rightarrow y = ce^{\cancel{\frac{1}{3(y/x)^3}}}$ $\Rightarrow$ $\boxed{y = ce^{\frac{1}{3(y/x)^3}}}$

Q ① $(x^2 - 2y^2)dx + xydy = 0$

The ~~soln~~ eqn can be written as

$$\frac{dy}{dx} = -\left(\frac{x^2 - 2y^2}{xy}\right)$$

[$x = xt$
$y = yt$
$\frac{x^2t^2 - 2y^2t^2}{xt\,yt}$
$= \frac{\cancel{t^2}(x^2 - 2y^2)}{\cancel{xt \cdot yt}\ xy\cancel{t^2}}$
$= \frac{x^2 - 2y^2}{xy}$]

Putting $y = vx$

$$\frac{dy}{dx} = v\frac{dx}{dx} + x\frac{dv}{dx}$$

$$= v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = -\left(\frac{x^2 - 2v^2x^2}{x\,vx}\right)$$

$$\Rightarrow v + x\frac{dv}{dx} = -\frac{x^2(1 - 2v^2)}{x\,vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = -\frac{x^2(1 - 2v^2)}{x^2 v}$$

$$\Rightarrow v + x\frac{dv}{dx} = -\frac{1 - 2v^2}{v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1-2v^2}{v} - v$$

$$= \frac{-1+2v^2-v^2}{v}$$

$$\Rightarrow x\frac{dv}{dx} = \cancel{\frac{1-3x^2}{v}}\ \frac{v^2-1}{v}$$

By VSM $= \frac{v}{v^2-\cancel{1}}dv = \frac{dx}{x}$

Integrating both side

$$\Rightarrow \int \frac{v}{v^2-1}dv = \int \frac{dx}{x}$$

$$\Rightarrow \int \frac{v}{v^2-1}dv = \ln x + \ln C$$

Let $v^2 - 1 = t$

$$2v = \frac{dt}{dv}$$

$$\Rightarrow v\,dv = \frac{dt}{2}$$

$$\Rightarrow \int \frac{dt}{2t} = \ln x + \ln C$$

$$\Rightarrow \frac{1}{2}\ln t = \ln xC$$

$$\Rightarrow \ln t = \ln x^2C^2$$

$$\Rightarrow t = x^2C^2$$

$$\Rightarrow v^2 - 1 = x^2C^2$$

$$\Rightarrow \frac{y^2}{x^2} - 1 = x^2C^2$$

$$\Rightarrow \frac{y^2-x^2}{x^2} = x^2C^2$$

$$\Rightarrow y^2 - x^2 = x^4C^2$$

$$\Rightarrow y^2 = \cancel{x^4+Cx^2}\ x^2 + x^4C^2$$

H/W

(1) $x^2\frac{dy}{dx}-3xy-2y^2=0$

(2) $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$

(3) $2xy\,dy-(y^2-x^2)\,dx=0$

(1) $x^2\frac{dy}{dx}-3xy-2y^2=0$

a) $x^2\frac{dy}{dx}=3xy-2y^2$

The eqn can be written as :-

$$\frac{dy}{dx}=\frac{3xy-2y^2}{x^2}$$

Putting $y=vx$

$$\cancel{\frac{dy}{dx}=\frac{3xy-2y^2}{x^2}}$$

$$\frac{dy}{dx}=v\frac{dx}{dx}+x\frac{dv}{dx}$$

$$=v+x\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=\frac{3vx^2+2v^2x^2}{x^2}$$

$$\Rightarrow v+x\frac{dv}{dx}=\frac{x^2(3v+2v^2)}{x^2}$$

$$\Rightarrow v+x\frac{dv}{dx}=3v+2v^2$$

$$\Rightarrow x\frac{dv}{dx}=3v+2v^2-v$$

$$\Rightarrow x\frac{dv}{dx}=2v^2+2v$$

$$\Rightarrow x\frac{dv}{dx}=2v(v+1)$$

$x=xt$

$y=yt$

$$=\frac{3xt\cdot yt-2y^2t^2}{x^2t^2}$$

$$=\frac{3xyt^2-2y^2t^2}{x^2t^2}$$

$$=\frac{t^2(3xy-2y^2)}{x^2t^2}$$

$$=\frac{3xy-2y^2}{x^2}$$

By VSM $= 2(v)(v+1)$

$\Rightarrow x\frac{dv}{dx} = 2(v)(v+1)$

Integreating both side

$\Rightarrow \int \frac{dv}{v(v+1)} = \int \frac{2dx}{x}$

$\Rightarrow \int \frac{1}{v} - \frac{1}{(v+1)} dv = 2\log x + \log c$

$\Rightarrow \log v - \log(v+1) = \log x^2 + \log c$

$\Rightarrow \cancel{\log}\left(\frac{v}{v+1}\right) = \cancel{\log}(cx^2)$

$\Rightarrow \frac{v}{v+1} = cx^2$

$\frac{\frac{y}{x}}{\frac{y}{x}+1} = cx^2$

$\Rightarrow \frac{y}{x+y} = cx^2$

$\boxed{y = cx^2(x+y)}$

(2) $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$

$\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$

Let $y = vx \Rightarrow v = \frac{y}{x}$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = v + \tan v$

$\Rightarrow x\frac{dv}{dx} = \cancel{v} + \tan v - \cancel{v}$

$\Rightarrow x\frac{dv}{dx} = \tan v$

By VSM $= \tan v$

$\Rightarrow x\frac{dv}{dx} = \tan v$

Integreating both side

$\Rightarrow \int \frac{dv}{\tan v} = \int \frac{dx}{x}$

$\Rightarrow \int \cot v \, dv = \ln x + c$

$\Rightarrow -\log|\operatorname{cosec}(v)| = \log x + \log c$

$\Rightarrow \log|\sin(v)| = \log(x \cdot c)$

$\Rightarrow \sin(v) = xc$

$\Rightarrow \sin y/x = x \cdot c$ Ans.

(3) $2xy\,dy - (y^2 - x^2)dx = 0$

$\Rightarrow 2xy\,dy - (y^2 - x^2)dx = 0$

$\Rightarrow 2xy\,dy = (x^2 - y^2)dx$

$\Rightarrow \frac{dy}{dx} = \frac{x^2 - y^2}{-2xy}$

$\Rightarrow$

$$\left[\begin{array}{l} x = xt \\ y = yt \\ = \frac{y^2t^2 - x^2t^2}{2xt \cdot yt} \\ = \frac{\cancel{t^2}(y^2 - x^2)}{2xy\cancel{t^2}} \\ = \frac{x^2 - y^2}{2xy} \end{array}\right.$$

Putting $y = vx$

$x = v/y$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{x^2 - v^2x^2}{-2x(vx)}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{\cancel{x^2}(1 - v^2)}{-\cancel{x^2}(2v)}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{1 - v^2}{-2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1 - v^2}{-2v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{-2v}$

$= \frac{-(1 + v^2)}{-2v}$

By vsm $= \frac{2v}{(1+v^2)}dv = \frac{dx}{x}$

Integrating both side

$= \int \frac{2v}{1+v^2}dv = -\int \frac{dx}{x}$

Let $(1+v^2) = t$

$\Rightarrow 2v = \frac{dt}{dv}$

$\Rightarrow v\,dv = \frac{1}{2}dt$

$\Rightarrow \ln t = -\ln x + \ln c$

$\ln t = \ln c/x$

$\Rightarrow \ln(1+v^2) + \ln x = \ln v$

$\Rightarrow \ln t = -\ln x + \ln c$

$\Rightarrow \ln(t \cdot x) = \ln c$

$\Rightarrow tx = c$

$\Rightarrow (v^2+1)x = c$

$\Rightarrow \left(\frac{y^2}{x^2}+1\right)x = c$

$\Rightarrow \left(\frac{y^2+x^2}{x^2}\right)x = c$

$\Rightarrow y^2+x^2 = xc$

$\Rightarrow \boxed{y^2 = xc - x^2}$ Ans.

$\Rightarrow \cancel{y^2 = x^4c^2 - x^2}$

Date-24.11.22

Special forms of Homogeneous eqn:-

$$\frac{dy}{dx} = \frac{ax+by+c}{a'x+b'y+c'}$$

where $\frac{a}{a'} \neq \frac{b}{b'}$

→ Lines $ax+by+c=0$ &

$a'x+b'y+c'=0$ are intersecting

→ if (h,k) be the point of their intersection then we have proceeding.

Q. $\frac{dy}{dx} = \frac{x+y+4}{x-y-6}$

We put $x = X+h \Rightarrow dx = dX + 0$

$y = Y+K \Rightarrow dy = dY + 0$

$$\frac{dY}{dX} = \frac{X+h+Y+K+4}{X+h-Y-K-6}$$

$$\Rightarrow \frac{dY}{dX} = \frac{(X+Y)+(h+K+4)}{(X-Y)+(h-K-6)}$$

Let $\frac{dY}{dX} = \frac{X+Y}{X-Y}$

Let $= h+K+4 = 0$

$h-K-6 = 0$

$2h - 2 = 0$

$h = 1 \quad K = 5$

$h+K=4$

$h-K=6$

$h = 1$

$K = -5$

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

Let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x+y}{x-y}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x+vx}{x-vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x(1+v)}{x-vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \left(\frac{1+v}{1-v}\right)$$

$\Rightarrow x\frac{dv}{dx} = \left(\frac{1+v}{1-v}\right) - v$

$= \frac{1+v - v(1-v)}{1-v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+v-v+v^2}{1-v} = \frac{1+v^2}{1-v}$

By VSM $= \cancel{\frac{1+v^2}{1-v}dv = \frac{dx}{x}}$ $\left(\frac{1-v}{1+v^2}\right)dv = \frac{dx}{x}$

Integrating both side

$= \int\frac{dx}{x}$

$\Rightarrow \int\frac{1-v}{1+v^2}dv = \int\frac{dx}{x}$

$\Rightarrow \int\frac{1-v}{1+v^2}dv = \int\frac{dx}{x}$

$\Rightarrow \int\frac{1}{1+v^2}dv - \int\frac{v}{1+v^2}dv = \int\frac{dx}{x}$

$\Rightarrow \tan^{-1}v - \frac{1}{2}\ln(1+v^2) = \ln x + C$

$1+v^2 = t$

$2v = \frac{dt}{dv}$

$\Rightarrow v\,dv = \frac{dt}{2}$

$\Rightarrow \tan^{-1}v = \ln\sqrt{1+v^2}\,x + C$

$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \ln\sqrt{1+\frac{y^2}{x^2}}\,x + C$

$\Rightarrow \tan^{-1}\frac{y}{x} = \ln\sqrt{\frac{x^2+y^2}{x^2}}\,x + C$

$= \ln\frac{1}{x}\sqrt{x^2+y^2}\,x + C$

$= \ln\sqrt{x^2+y^2} + C$

$x = X+h$

$y = Y+K$

$X = x-h$

$Y = y-K$

$\Rightarrow \tan^{-1}\frac{(y-K)}{(x-h)} = \ln\sqrt{(x-h)^2+(y-K)^2} + C$

$\Rightarrow \tan^{-1}\left(\frac{y+5}{\ }\right) = \ln\sqrt{(x-1)^2+(y+5)^2} + C$

HW

(1) $$\frac{dy}{dx} = \frac{x-y+1}{x+y-3}$$

(2) $$(2x+y+3)\frac{dy}{dx} = (x+2y+3)$$

(1) $$\frac{dy}{dx} = \frac{x-y+1}{x+y-3}$$

We put $x = X+h \Rightarrow dx = dX+h$

$y = Y+K \Rightarrow dy = dY+K$

$$\Rightarrow \frac{dy}{dx} = \frac{x+h-\cancel{y}-h+1}{x+h+y+h-3}$$

$$= \frac{(x-y)\,1}{(x+y)\,(2h-3)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x-y}{x+y}$$

Let $h+x+1=0$, $h-k-3=0$

$\cdot h = 1 \quad k = 2$

$$\left[\begin{array}{l} h+x=-1 \\ h-x=3 \\ \hline 2h=2 \\ h=1 \\ 1-x=3 \\ -x=3-1 \\ x=-2 \end{array}\right.$$

$$\Rightarrow \frac{dy}{dx} = \frac{X-Y}{X+Y}$$

Let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x-y}{x+y}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x-vx}{x+vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{\cancel{x(1+v)}}{} \; \frac{x-vx}{x(1+v)}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x(1-v)}{x(1+v)}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1-v}{1+v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1-v}{1+v} - v$$

$$= \frac{1-v-v-v^2}{1+v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1-2v-v^2}{1+v}$$

By VSM $\frac{1+v}{1-2v-v^2}\,dv = \frac{dx}{x}$

Integreating both side

$$\Rightarrow \int \frac{1+v}{1-2v-v^2}\,dv = \int \frac{dx}{x}$$

$$\Rightarrow \cancel{\int \frac{1}{1-v^2}\,dv + \int \frac{v}{1-v^2}\,dv} = \int \frac{dx}{x}$$

$\cancel{\Rightarrow \int \frac{1}{1-v^2}\,dx}$ $\Rightarrow$ Let $t = 1-2v-v^2$

$$\Rightarrow \frac{dt}{dv} = -2-2v$$

$$= -2(1+v)$$

$$\Rightarrow \frac{dt}{-2} = (1+v)\,dv$$

$$\Rightarrow -\frac{1}{2}\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2}\ln t = \ln x + \ln c$$

$$\Rightarrow -\frac{1}{2}\ln t = 2\ln(xc)$$

$$\Rightarrow -\ln t = \ln(xc)^2$$

$$\Rightarrow \ln(xc)^2 + \ln t = 0$$

$$\Rightarrow x^2c^2 t = 0$$

$$\Rightarrow x^2c^2(1-2v-v^2) = e^0$$

$$\Rightarrow x^2c^2\left(1 - \frac{2y}{x} - \frac{y^2}{x^2}\right) = 1$$

$$\Rightarrow x^2 - 2xy - y^2 = \frac{1}{c^2}$$

or $x^2 - 2xy - y^2 + 2x + 6y = C$

$\Rightarrow (x-1)^2 - 2(x-1)(y-2) - (y$

$= 1/c$

(2) $(2x+y+3)\frac{dy}{dx} = (x+2y+3)$ $\quad$ or $(x-y)^3 = c(x+y+2)$

$\Rightarrow \frac{dy}{dx} = \frac{x+2y+3}{2x+y+3}$

Let $x = X+h \Rightarrow dx = dX+0$

$y = Y+K \Rightarrow dy = dY+0$

$\therefore \frac{dy}{dx} = \frac{dY}{dX}$

$\therefore \frac{dy}{dx} = \left(\frac{x+h+2y+2K+3}{2x+2h+Y+K+3}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{x+2y+h+2K+3}{2x+Y+2h+K+3}$

Let $h+2K+3 = 0$ —(i)

$2h+K+3 = 0$ —(ii)

Eqn (i) $\times 2 = 2h+4K+6 = 0$

Eqn (ii) $\times 1 = 2h+K+3 = 0$

$3K+3 = 0$

$K = -1$

$\therefore 2h+1+3 = 0$

$h = -1$

$\therefore \frac{dy}{dx} = \frac{x+2Y}{2x+Y}$

Let $Y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{x+2vx}{2X+vX}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{1+2v}{2+v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+2v}{2+v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{1+2v-2v-v^2}{2+v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1-v^2}{2+v}$

$\Rightarrow \frac{2+v}{1-v^2}\,dv = \frac{dx}{x}$ (By VSM)

$\Rightarrow \int \frac{2}{1-v^2}\,dv + \int \frac{v\,dv}{1-v^2} = \int \frac{dx}{x}$

$\Rightarrow 2\times\frac{1}{2} \ln\left(\frac{1+v}{1-v}\right) - \frac{1}{2}\ln(1-v^2) = \ln x + \ln c$

$\Rightarrow \ln\left(\frac{1+v}{1-v}\right) - \ln\sqrt{1-v^2} = \ln(x\cdot c)$

$\Rightarrow \left(\frac{x+y}{x-y}\right) \times \frac{\sqrt{x^2-y^2}}{x} = xc$

$\Rightarrow 1+v\ \ln\left(\frac{1+v}{1-v}\times\frac{1}{\sqrt{1-v^2}}\right) = \ln(x\cdot c)$

$\Rightarrow \left(\frac{1+\frac{y}{x}}{1-\frac{y}{x}} + \frac{1}{\sqrt{1-\frac{y^2}{x^2}}}\right) = xc$

$\Rightarrow \frac{(x+y)}{x-y}\cdot\left(\frac{x}{x^2-y^2}\right) = x\cdot c$

$\frac{1}{x}$

$-\frac{1\cdot x - x\cdot}{x^2}$ $\Rightarrow (x+y) = (x-y)\sqrt{x^2-y^2}$

$\Rightarrow (x+2)+(y-1) = (x+2-y+1)\sqrt{(x+1)^2 (y+1)^2} + c$

$\Rightarrow (x+y+1) = (x-y+\cancel{3})\sqrt{(x+1)^2 - (y+1)^2} + c$

$\Rightarrow (x-y)^3 = (x+y+z)$

Date - 25/11/22

*Exact Differential equation:-

→ The differential equation $M(x,y)dx + N(x,y)dy = 0$ is ~~exact~~ called exact diff eqn.

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Step-1

- give a differential eqn we have find out $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Step-2

$\int M(x,y)\,dx +$ — y as constant

$\int N(x,y)\,dy$ — only those terms which do not contain x. If x term is present fgn = zero.

Ex- Solve the differential eqn.

$$(y\cos x + \sin y + y)\,dx + (\sin x + x\cos y + x)\,dy = 0$$

Here

$M = y\cos x + \sin y + y$

$N = \sin x + x\cos y + x$

$$\frac{\partial M}{\partial y} = \frac{\partial(y\cos x + \sin y + y)}{dy}$$

$= \cos x \frac{\partial y}{\partial y} + \cos y + 1$

$= \cos x + \cos y + 1$

$\frac{\partial N}{\partial x} = \cos x + \cos y + 1$

Hence $\frac{\partial m}{\partial y} = \frac{\partial N}{\partial x}$

Thus the differential equation is exact.

$\therefore \int M\,dx + \int N\,dy = c$

$= \cancel{\int (\cos x + \cos y + 1)dx + \int (\cos x + \cos y + 1)\,dy = c}$

$\Rightarrow \int (y\cos x + \sin y + y)\,dx + \int (\sin x + x\cos y + x)\,dy = c$

$\Rightarrow y\sin x + x\sin y + xy + 0 = c$

$\Rightarrow y\sin x + x\sin y + xy = c$

Q.2

$- (e^y dx + (xe^y + 2y)\,dy = 0$

Here ~~M = ...~~

~~...~~

Here $M = e^y$

$N = xe^y + 2y$

$\frac{\partial m}{\partial y} = e^y$

$\frac{\partial N}{\partial x} = xe^y + 2y$

$= e^y$

Hence $\frac{\partial m}{\partial y} = \frac{\partial N}{\partial x}$

Thus the differential equation is exact

$\therefore \int M\,dx + \int N\,dy = c$

$\Rightarrow \int e^y + \boxed{\int xe^y + 2y = c}$

$\Rightarrow e^y \int dx + \int 2y\,dy = c$

$\Rightarrow x e^y + \frac{2y^2}{2} = c$

$\Rightarrow xe^y + y^2 = c$

③ $xe^{x^2+y^2}dx + y(e^{x^2+y^2}+1)dy = 0$

Here

$M = xe^{x^2+y^2}$

$N = y(e^{x^2+y^2}+1)$

$\frac{\partial M}{dy} = xe^{x^2+y^2}$ $\Rightarrow \frac{d\,xe^{x^2+y^2}}{dy}$ $\frac{d}{dy}(xe^{x^2+y^2})$

$= e^{x^2+y^2}$ $\Rightarrow \cancel{e^{x^2}+} = x\frac{\partial e}{dy}(x^2+y^2)+0$

$\frac{dN}{dx} = y(e^{x^2+y^2}+1)$ $\Rightarrow 2y = 2xye^{x^2}e^{y^2}$

$= e^{x^2+y^2}+1$

Thus differential eqn is not exact.

$\therefore \int M dx + \int N dy = C$

$\frac{\partial N}{\partial x} = ye^{x^2+y^2}2x$

$= 2xye^{x^2}e^{y^2}$

$\therefore \int M dx + \int N dy = 0$

$\Rightarrow \int xe^{x^2}e^{y^2}dx + \int y(e^{x^2}e^{y^2}+y)dy = 0$

$\Rightarrow \int xe^{x^2}e^{y^2}dx + \int (ye^{x^2}e^{y^2}+y\,dy = 0$

Let $x^2+y^2 = t$

$\Rightarrow 2x = \frac{dt}{dx}$

$\Rightarrow x\,dx = \frac{dt}{2}$

$\Rightarrow \int e^t\frac{dt}{2} + \frac{y^2}{2} = C$

$\Rightarrow \frac{1}{2}e^t + \frac{1}{2}y^2 = C$

$\Rightarrow \frac{1}{2}e^{x^2+y^2} + \frac{1}{2}y^2 = C$

$\Rightarrow \frac{1}{2}e^{0+0} + \frac{1}{2}0 = C$

$\Rightarrow \frac{1}{2}\cdot 1 = C$

$\Rightarrow C = \frac{1}{2}$ $\boxed{\Rightarrow e^{x^2+y^2}+y^2 = 1}$

$\Rightarrow \frac{1}{2}e^{x^2+y^2} + \frac{1}{2}y^2 = \frac{1}{2}$

Q. $e^{x^2+y^2} + y^2 = 1$

date- 27/11/22

(2) $(x+y+1)dx + (x-y^2+3)dy = 0$

(3) $(\sin x + \tan y + 1)dx + (\cos x \cdot \sec^2 y) \cdot dy = 0$

(4) $x\,dx + y\,dy + \frac{x\,dy - y\,dx}{x^2+y^2} = 0$

HW

(1) $(x+y+1)dx + (x-y^2+3)dy = 0$

$M = x+y+1$

$\frac{\partial M}{\partial y} = 1$

$N = x-y^2+3$

$\frac{\partial N}{\partial x} = 1$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\because \int M\,dx + \int N\,dy = C$

$\Rightarrow \int (x+y+1)dx + \int (x-y^2+3)\,dy = C$

$\Rightarrow \left(\frac{x^2}{2} + 0 + x\right) dx + \int 0 - \frac{y^3}{3} + 3y) = C$

$\Rightarrow \frac{x^2}{2} + x - \frac{y^3}{3} + 3y = C$

Q2. $(\sin x + \tan y + 1)dx + (\cos x \cdot \sec^2 y)dy = 0$

$M = \sin x + \tan y + 1$ ——(1)

$\Rightarrow \frac{\partial M}{\partial y} = \sec^2 y \sin x$

$N = \cos x \cdot \sec^2 y$ ——(1)

$\frac{\partial N}{\partial y} = -\sin x \cdot \sec^2 y$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial y}$

The required eqn is not exact.

Date - 28 NOV

Partial fraction :-

(1) $\dfrac{px+q}{(x-a)(x-b)}, \ a \neq b \rightarrow \dfrac{A}{x-a} + \dfrac{B}{x-b}$

(2) $\dfrac{px+q}{(x-a)^2} \rightarrow \dfrac{A}{x-a} + \dfrac{B}{(x-a)^2}$

(3) $\dfrac{px^2+qx+r}{(x-a)(x-b)(x-c)} \longrightarrow \dfrac{A}{x-a} + \dfrac{B}{x-b} + \dfrac{C}{x-c}$

(1) $\displaystyle\int \frac{x}{x^2-11x+24}\,dx$

$= \displaystyle\int \frac{x}{x^2-8x-3x+24}\,dx$

$= \displaystyle\int \frac{x}{(x-3)(x-8)}\,dx$ - (1st form)

(4) $\dfrac{px^2+qx+r}{(x-a)^2(x-b)}$

$= \dfrac{A}{x-a} + \dfrac{B}{(x-a)^2} + \dfrac{C}{x-b}$

(5) $\dfrac{px^2+qx+r}{(x-a)(x^2+bx+c)}$

$= \dfrac{A}{x-a} + \dfrac{Bx+C}{x^2+bx+c}$

Ex:- 1

(1) $\displaystyle\int \frac{dx}{(x+1)(x+2)}$

$\therefore$ Let $\dfrac{1}{(x+1)(x+2)} = \dfrac{A}{x+1} + \dfrac{B}{x+2}$

Let $\dfrac{1}{\cancel{(x+1)}(x+2)} = \dfrac{A(x+2)+B(x+1)}{\cancel{(x+1)}(x+2)}$

$\Rightarrow 1 = A(x+2) + B(x+1)$

$= Ax + 2A + Bx + B$

$= Ax + Bx + 2A + B$

$1 = x(A+B) + 2A + B$

Equating / comparing co-efficient x and ~~Equat~~ constant term in both side.

$A + B = 0 \longrightarrow (i)$

$2A + B = 1 \longrightarrow (ii)$

From eqn (i) and (ii)

$$\begin{array}{r} A + B = 0 \\ \underset{(-)}{2A} \underset{(-)}{+B} = \underset{(-)}{1} \\ \hline \end{array}$$

$\Rightarrow -A = -1$

$\Rightarrow A = 1$

$\Rightarrow 1 + B = 0$

$\Rightarrow B = -1$

$\therefore$

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$\Rightarrow \frac{1}{(x+1)(x+2)} = \frac{1}{x+1} + \frac{-1}{x+2}$$

Integrating both side

$$\int \frac{dx}{(x+1)(x+2)} = \int \frac{dx}{x+1} - \int \frac{dx}{x+2}$$

$= \ln(x+1) - \ln(x+2)$

$= \ln\left|\frac{x+1}{x+2}\right| + C$

Q. $\frac{8x+5}{x^2-13x+36}\,dx$

$= \frac{8x+5}{x^2-9x-4x+36}\,dx$

$= \frac{8x+5}{x(x-9)-4(x-9)}\,dx$

$= \frac{8x+5}{(x-4)(x-9)}$

$\Rightarrow \frac{8x+5}{(x-4)(x-9)} = \frac{A}{x-4} + \frac{B}{x-9}$

$\Rightarrow \frac{8x+5}{(x-4)(x-9)} = \frac{A(x-9)+B(x-4)}{(x-4)(x-9)}$

$\Rightarrow \frac{8x+5}{(x-4)(x-9)} = \frac{A(x-9)+B(x-4)}{(x-4)(x-9)}$

$\Rightarrow 8x+5 = A(x-9)+B(x-4)$

$\Rightarrow 8x+5 = Ax-9A+Bx-4B$

$\Rightarrow 8x+5 = Ax+Bx-9(x+4)$

Equating co-efficients x and constant in both side.

$A+B=8$

$-4A-9B=5$

$A+B = 8+\frac{37}{5}$

$= \frac{40+37}{5}$

$= \frac{77}{5}$

$\Rightarrow \frac{A}{x-9} + \frac{B}{x-4}$

$\Rightarrow \frac{77}{5}/x-9 + \frac{33/5}{x-4}$

$\Rightarrow \frac{77}{5}\log(x-9) - \frac{33}{5}\log(x-4)$

(3) $x\,dx + y\,dy + \frac{x\,dy - y\,dx}{x^2+y^2} = 0$

$= (x^2+y^2)\,x\,dx + (x^2+y^2)\,y\,dy + x\,dy - y\,dx = 0$

$\Rightarrow (x^2+y^2)\,x\,dx - y\,dx + (x^2+y^2)\,y\,dy + x\,dy = 0$

$\Rightarrow \{(x^2+y^2)x - y\}\,dx + \{(x^2+y^2)\,y + x\}\,dy = 0$

$\frac{\partial M}{\partial y} = 2xy - 1$

$\frac{\partial N}{\partial x} = 2xy + 1$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ (It is not exact)

---

* Find $\int \frac{x^2}{(x^2+1)(x^2+4)}\,dx$, Let $y = x^2 \Rightarrow \frac{y}{(y+1)(y+4)}$ first form.

(1) $\int \frac{4x+1}{(x+1)}\,dx$

(2) $\int \frac{x+4}{x^2-x-2}\,dx$

(3) $\int \frac{x-25}{x^2+5x-24}\,dx$

(4) $\int \frac{1}{(x^4-1)}\,dx$

(1) Find $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$

a) $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$

Let $y = x^2$

$$\frac{y}{(y+1)(y+4)} = \frac{A}{(y+1)} + \frac{B}{(y+4)}$$

$$\Rightarrow \frac{y}{(y+1)(y+4)} = \frac{Ay+4A+By+B}{(y+1)(y+4)}$$

$$\Rightarrow y = Ay + By + 4A + B$$

$$\Rightarrow y = (A+B)y + 4A + B$$

$$\Rightarrow y + 0 = (A+B)y + 4A + B$$

$$\therefore A + B = 1$$

$$4A + B = 0$$

$$-3A = 1$$

$$A = -\frac{1}{3} \Rightarrow B = 1 - A$$

$$= 1 + \frac{1}{3}$$

$$= 4/3$$

$$\therefore \frac{y}{(y+1)(y+4)} = \frac{-\frac{1}{3}}{y+1} + \frac{4/3}{y+4}$$

$$\Rightarrow \int \frac{y}{(y+1)(y+4)} dx = \int \frac{(-1/3)}{y+1} + \frac{4/3}{y+4} dx$$

$$\Rightarrow \int \frac{x^2}{(x^2+1)(x^2+4)} dx = \int \left(\frac{-1/3}{x^2+1} + \frac{4/3}{\cancel{y}\, x^2+4} \right) dx$$

$$= -\frac{1}{3} \tan^{-1} x + \frac{4}{3} \cdot \frac{1}{2} \tan^{-1} \frac{x}{2}$$

$$= -\frac{1}{3} \tan^{-1} x + \frac{2}{3} \tan^{-1} \frac{x}{2} + C$$

(2) $\int \frac{4x+1}{(x+1)(x-2)} dx$

$\therefore \frac{4x+1}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)}$

$\Rightarrow \frac{4x+1}{(x+1)(x-2)} = \frac{Ax-2A+Bx+B}{(x+1)(x-2)}$

$\Rightarrow 4x+1 = (A+B)x - 2A + B$

$\therefore A+B = 4$
$-2A+B = 1$

$\Rightarrow 3A = 3$

$A = 1$ & $B = 3$

$\therefore \int \frac{4x+1}{(x+1)(x-2)} dx$

$= \int \left(\frac{1}{x+1} + \frac{3}{x-2}\right) dx$

$\Rightarrow \int \frac{4x+1}{(x+1)(x-2)} dx = \ln(x+1) + 3\ln(x-2) + C$

Q3. $\int \frac{x+4}{x^2-x-2} dx = \int \frac{x+4}{x^2-2x+x-2} dx$

$= \int \frac{x+4}{x(x-2)+1(x-2)} dx$

$= \int \frac{x+4}{(x+2)(x+2)} dx$

$\Rightarrow \frac{x+4}{\cancel{(x+1)(x-2)}\ (x-2)(x+1)} dx = \frac{A}{x-2} + \frac{B}{x+1}$

$= \frac{Ax+A+Bx-2B}{(x-2)(x+1)}$

$\Rightarrow \frac{x+4}{(x-2)(x+1)} = \frac{(A+B)x + A - 2B}{(x-2)(x+1)}$

$\Rightarrow x+4 = (A+B)x + A - 2B$

$\therefore A+B=1$

$A-2B=4$

$\Rightarrow 3B=-3$

$B=-1$

$A=1-B$

$=1+1$

$=2$

$\Rightarrow \int \frac{x+4}{(x-2)(x+1)}\,dx$

$\Rightarrow \int \left(\frac{2}{(x-2)}+\frac{-1}{(x+1)}\right)dx$

$\Rightarrow 2\ln(x-2)-\ln(x+1)+C$

4. $\int \frac{x-25}{x^2+5x-2x}\,dx =$

$= \int \frac{x-25}{x^2+8x-3x-2x}\,dx$

$= \int \frac{x-25}{(x+8)(x-3)}\,dx$

$\therefore \frac{x-25}{(x+8)(x-3)} = \frac{A}{x+8}+\frac{B}{x-3}$

$\Rightarrow \frac{x-25}{(x+8)(x-3)} = \frac{Ax-3A+Bx+8B}{(x+8)(x-3)}$

$\Rightarrow x-25 = (A+B)x-3A+8B$

$\therefore A+B=1$

$-3A+8B=-25$

$\Rightarrow A=1-B$

$-3A+8B=-25$

$\Rightarrow -3(1-B)+8B=-25$

$\Rightarrow 11B=-22$

$\Rightarrow B=-2$

$A=3$

$\int \frac{x-25}{(x+8)(x-3)}\,dx = \int \left(\frac{3}{x+8}-\frac{2}{x-3}\right)dx$

$= 3\ln(x+8)-2\ln(x-3)+c$

$= \ln(x+8)^3-\ln(x-3)^2+c$

$= \ln\frac{(x+8)^3}{(x-3)^2}+c$

Q5) $\int \frac{dx}{x^4-1}$

$= \int \frac{dx}{(x^2)^2-(1)^2}$

$= \int \frac{dx}{(x^2-1)(x^2+1)}$

let $y = x^2$

$\int \frac{dx}{(y-1)(y+1)}$

$\Rightarrow \frac{1}{(y-1)(y+1)} = \frac{A}{y-1} + \frac{B}{y+1}$

$\Rightarrow \frac{1}{(y-1)(y+1)} = \frac{Ay+A+By-B}{(y-1)(y+1)}$

$\Rightarrow 1 = (A+B)y + A - B$

$\therefore A+B = 0$

$A-B = 1$

$2A = 1$

$A = 1/2$

$B = -1/2$

$\therefore \int \frac{dx}{(y-1)(y+1)} = \int \left(\frac{\frac{1}{2}}{y-1} + \frac{-\frac{1}{2}}{y+1}\right) dx$

$\Rightarrow \int \frac{dx}{(x^2-1)(x^2+1)} = \int \left(\frac{\frac{1}{2}}{x^2-1} - \frac{1/2}{x^2+1}\right) dx$

$\Rightarrow \int \frac{dx}{(x^2-1)(x^2+1)} = \int \left(\frac{1/2}{x^2-1} - \frac{1/2}{x^2+1}\right) dx$

$\Rightarrow \int \frac{dx}{(x^2-1)(x^2+1)} = \frac{1}{2}\ln(x^2-1) - \frac{1}{2}\ln(x^2+1) + C$

Date- 29/11/22

Linear differential equation:-

A differential equation of the form

$$\frac{dx}{dy} + P(y)x = Q(y) \quad \text{--- (1)}$$
on (x)

or $\frac{dy}{dx} + P(x)\, y = Q(x)$ --- (11)
on (y)

is called linear differential equation with 1 order and 1 degree.

$I.F = e^{\int P(x)dx}$

$y \cdot IF = \int Q(x) \cdot I.F\, dx + c$

Step-1

linear x $P(x) = ?$
$Q(x) = ?$

Step-2

$I.F = e^{\int P(x)dx}$

$Y = e^{\int P(y)dy}$

Step-3

General soln

$y \cdot IF = \int Q(x) \cdot IF\, dx + c$

$x \cdot IF = \int Q(y) \cdot I.F\, dy + c$

For Example- $x\frac{dy}{dx} - (x+1) y = x^2 - \cancel{x^3}$

$\Rightarrow \frac{dy}{dx} - \left(\frac{x+1}{x}\right) y = x - x^2$ (dividing x both side)

$P(x) = \left(-\frac{x+1}{x}\right)$ $Q(x) = x - x^2$

$I.f = e^{\int P(x)dx}$

$= e^{-\int \frac{(x+1)}{x} dx}$

$= e^{\left(\frac{x+1}{x}\right)}$

$\Rightarrow e^{\left(\frac{x}{x} + \frac{1}{x}\right)}$

$= e^{1 - \frac{1}{x} dx}$

$= e^{-x - \ln x}$

$= e^{-x} e^{\ln \frac{1}{x}}$

$* \; e^{\ln(x)} = x$

$e^{\ln(x^2)} = x^2$

$e^{\ln(1+x)} = (1+x)$

I·H

$I.F = e^{-x} \cdot \frac{1}{x}$

General soln

$y\, I.F = \int Q(x).\, If\, dx$

$y e^{-x} \frac{1}{x} = \int (x - x^2)\, e^{-x} \frac{1}{x}\, dx$

$\Rightarrow \frac{y}{x} e^{-x} = \int (1-x)\, e^{-x}\, dx$

$= \int e^{-x} dx - \int x e^{-x} dx$

$= -e^{-x} - [x - e^{-x} + \int e^{-x} \cdot 1]$

$= -e^{-x} - [-xe^{-x} - e^{-x}] + c$

$= -\cancel{e^{-x}} + xe^{-x} + \cancel{e^{-x}} + c$

$= xe^{-x} + c$

$\Rightarrow \frac{y}{x} e^{-x} = xe^{-x} + c$

$\Rightarrow y = x^2 + xce^{x}$

Solve $y' - 2y = 4$

$\Rightarrow \frac{dy}{dx} - 2y = 4$

$P(x) = 4$

$Q(x) = 4$

$If = e^{\int P(x) dx}$

$= e^{\int -2 dx}$

$= e^{-2x}$

GS: $y(e^{-2x}) = \int 4(e^{-2x}) dx$

$\Rightarrow y e^{-2x} = -4/2\, e^{-2x} + c$

$\Rightarrow ye^{-2x} = -2e^{-2x} + c$

$\Rightarrow y = -2 + e^{2x} c$

$y(0) = 0$

$\Rightarrow 0 = -2 + \cancel{e^{0}} c$

$\Rightarrow c = 1 \Rightarrow y = -2 + 2e^{2x}$ (Ans)

H.I

(1) $\frac{dy}{dx} - y\cot x = 2x\sin x$

(2) $x\log x \frac{dy}{dx} + y = 2\log x$

(3) $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

(4) $\frac{dy}{dx} + y\tan x = \sin 2x$ with $y(0) = 1$

$P(x) = y\tan x$

$Q(x) = \sin 2x$

Answers

Q1. $\frac{dy}{dx} - y\cot x = 2x\sin x$

Here $P(x) = -\cot x$

$Q(x) = 2x\sin x$

$If = e^{\int -\cot x}$

$= e^{-\int \cot x dx}$

$= e^{-\ln(\sin x)}$ $= \operatorname{cosec} x$

G.S

$y \operatorname{cosec} x = \int 2x\sin x \cdot \operatorname{cosec} x \, dx$

$\Rightarrow y\operatorname{cosec} x = \int 2x\,dx$

$\Rightarrow \frac{y}{\sin x} = \frac{2x^2}{2} + c$

$\Rightarrow y = x^2\sin x + \sin x \cdot c$

Q2. $x\log x \frac{dy}{dx} + y = 2\log x$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x\log x} = \frac{2}{x}$

Here $P(x) = \frac{1}{x\log x}$

$Q(x) = \frac{2}{x}$

$If = e^{\int \frac{1}{x\ln x} dx}$

$= e^{\int \frac{1}{t} dt}$ Let $t = \ln x$

$= e^{\ln(t)} = t$ $\Rightarrow \frac{dt}{dx} = \frac{1}{x}$

$= \ln x$ $\Rightarrow dt = \frac{dx}{x}$

G.S

$y(\ln x) = \int \frac{2}{x}\ln x \, dx$

$= 2\int \frac{\ln x}{x} dx$

$= 2\int u \, dx$

$= 2\frac{u^2}{2} + c$

$\Rightarrow u^2 + c$

$\Rightarrow y(\ln x) = (\ln x^2) + c$

$\Rightarrow y = \ln x + \frac{c}{\ln x}$

Q3. $(1+y^2)dx = (-\tan^{-1}y - x)dy$

$\Rightarrow \frac{dy}{dx} = \frac{1+y^2}{-\tan^{-1}y - x}$

$\Rightarrow \frac{dy}{dx} = \frac{-\tan^{-1}y - x}{1+y^2}$

$\Rightarrow \frac{dy}{dx} = -\frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$

$\Rightarrow \frac{dy}{dx} + \frac{x}{1+y^2} = -\frac{\tan^{-1}y}{1+y^2}$

$\therefore P(x) = \frac{1}{1+y^2}$

$Q(x) = -\frac{\tan^{-1}y}{1+y^2}$

$\therefore IF = e^{\int \frac{1}{1+y^2} dy}$

$= e^{\tan^{-1}x}$

$xe^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2} \times e^{\tan^{-1}y} dy$

$\Rightarrow dt = \frac{dy}{1+y^2}$

G.S

$\therefore xe^{\tan^{-1}y} = \int t \cdot e^t dt$

$= te^t - \int 1 \cdot e^t dt$

$= te^t - e^t + c$

$\Rightarrow xe^{\tan^{-1}y} = \tan^{-1}y \cdot e^{\tan^{-1}y} - e^{\tan^{-1}y} + c$

$\Rightarrow xe^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y - 1) + C$

Date- 1st dec 2022

Date - 1st december

Equation to liner form
(Bernoulli equation)

→ A equation written in the form $\frac{dy}{dx} + P(x)y = Q(x)y^n$ Bernoulli eqn.

y is liner.

working rule:-

(1) devide the eqn by y^n

$\Rightarrow y^{-n}[y^n \frac{dy}{dx} + P(x) y^{1-n} = Q(x)]$ ⟶ liner form

(2) Put $y^{-n+1} = t$ ~~$\frac{dy}{dx}$~~

(3) Now convert in to $\frac{dt}{dx} + (1-n)Pt = (1-n)Q$.

(4) Eqn which is a linear D.E.

(1) $x\frac{dy}{dx} + y = y^2 \log x$

$\Rightarrow \frac{dy}{dx} + \frac{1}{x}\cdot y = \frac{\log x}{x} y^2$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} + \frac{1}{x}\cdot\frac{y}{y^2} = \frac{\log x}{x}\frac{y^2}{y^2}$

$\Rightarrow y^{-2}\frac{dy}{dx} + \frac{1}{x}y^{-1} = \frac{\log x}{x}$

Let $y^{-1} = t$

$\Rightarrow y^{-1-1}\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow -y^{-2}\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow -y^{-2}\frac{dy}{dx} = -\frac{dt}{dx}$

$\Rightarrow y^{-2}\frac{dy}{dx} = -\frac{dt}{dx}$

$\Rightarrow -\frac{dt}{dx} + \frac{1}{x}t = \frac{\log x}{x}$

$\Rightarrow \frac{dt}{dx} - \frac{1}{x}t = -\frac{\log x}{x}$

$P(x) = -\frac{1}{x}$

$Q(x) = -\frac{\log x}{x}$

$I.f = \int e^{P(x)dx} = \int e^{-\frac{1}{x}dx} = e^{-\ln x} \Rightarrow x^{-1} \Rightarrow \frac{1}{x}$

General sollution

$I.F = \int Q(x) - I.F\, dx$

$\Rightarrow t\frac{1}{x} = \int -\frac{\log x}{x}\cdot\frac{1}{x}dx$

$= \int -\frac{\log x}{x^2}dx$

$= \int -\log x\, x^{-2}dx$

$t\frac{1}{x} = \int -\log x\, x^{-2}dx$

$= -\int \log x \int x^{-2}dx - \int \frac{d(-\log x)}{dx}\left(\int x^{-2}dx\right)dx$

$= -\log x\left(\frac{x^{-2+1}}{-2+1}\right) - \int -\frac{1}{x}\left(\frac{x^{-2+1}}{-2+1}\right)dx$

$= -\log x(-x^{-1}) + \int -\frac{1}{x}x^{-1}dx$

$= \log x\frac{1}{x} - \int x^{-2}dx$

$= \frac{\log x}{x} - \frac{x^{-1}}{-1} + C$

$= \log x/x + 1/x + C$

Q $\frac{dy}{dx} - y\sec x = y^2 \cos x \cdot \sin x$

$\frac{dy}{dx} - y\sec x = y^2 \cos x \cdot \sin x$ (dividing y^2)

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \frac{y}{y^2}\sec x = \frac{y^2}{y^2}\cos x \cdot \sin x$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\sec x = \cos x \cdot \sin x$

Let $t = 1/y \Rightarrow -y^{-2}\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$

$\therefore -\frac{dt}{dx} - t\sec x = \cos x \cdot \sin x$

$\Rightarrow \frac{dt}{dx} + t\cdot\sec x = -\cos x \cdot \sin x$

$P(x) = \sec x$

$Q(x) = -\cos x \cdot \sin x$

I.F $= e^{\int \sec x}$

$= e^{\ln(\sec x + \tan x)}$

$= \sec x + \tan x$

$\left(-\cos x \cdot \sin x\right)\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)$

G.S. $t(\sec x + \tan x) = \int (-\cos x \cdot \sin x)(\sec x + \tan x)$

$\Rightarrow t(\sec x + \tan x) = -\int (\sin x + \sin^2 x)\,dx$

$\Rightarrow t(\sec x + \tan x) = -\int \left(\sin x + 1 - \frac{\cos 2x}{2}\right)dx$

$= -\int \left(\sin x + \frac{1}{2} - \frac{\cos 2x}{2}\right)dx$

$= -\left(-\cos x + x/2 - \frac{\sin 2x}{4}\right) + c$

$= \cos x - \frac{x}{2} + \frac{\sin 2x}{4} + c$

$\Rightarrow t(\sec x + \tan x) = \cos x - x/2 + \frac{\sin 2x}{4} + c$ (Answer)

$\Rightarrow \frac{1}{t(\sec x + \tan x)} = \frac{1}{\cos x} - \frac{2}{x} + \frac{4}{\sin 2x} + c$

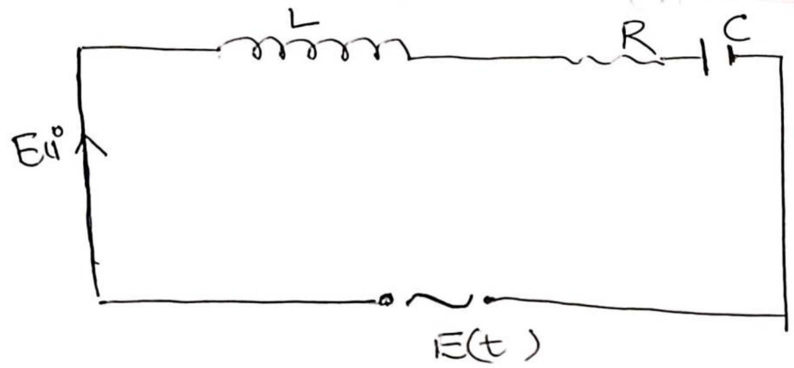
$y^{-1}\sec x+\tan x$

$$\Rightarrow \frac{y}{(\sec x+\tan x)} = \frac{1}{\cos x} - \frac{2}{x} + \frac{4}{\sin 2x} + c$$

$$\Rightarrow y = \frac{\sec x+\tan x}{\cos x} - \frac{2\sec x+2\tan x}{x} + \frac{4\sec x+4\tan x}{\sin 2x} + (\sec x+\tan x)+c$$

Date - 2nd dec

Application of electrical circuit :-



Considere the electric circuit as shown the figure.

where C → capacitance
L → inductance
R → Resistance
E(t) → voltage source
i(t) → current

By using Kirchhoff's law

$$\boxed{L\frac{di}{dt} + Ri = E(t)}$$

(L divided bothside)

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{E(t)}{L}$$

Q. When a resistance ~~~~ R is connected in series with an inductance L with constant emf of E volts. The current i at time t is given by

$$L\frac{di}{dt} + Ri = E$$ find the current at any time t if t = 0 and i = 0

Ans:- $$L\frac{di}{dt} + Ri = E \quad \text{---(i)}$$

Reducing eqn (i) we have (divide L both sides)

$$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L} \quad \text{---(ii)}$$

which is linear in i

Here $P(t) = R/L$

$Q(t) = E/L$

$$I.F = e^{\int P(t)\,d(t)}$$

$$= e^{\int R/L\,dt}$$

$$= e^{R/L\,t}$$

The required soln is

$$i.I.F = \int Q(t)$$

$$i.e^{R/L\,t} = \int \frac{E}{L}\cdot e^{R/L\,t}\,dt + c$$

$$= \frac{E}{L}\int e^{\frac{R}{L}t}\,dt + c$$

$$= \frac{E}{L}\frac{e^{\frac{R}{L}t}}{\frac{R}{L}} + c$$

$$= \frac{E}{L}\times\frac{L}{R}\times e^{\frac{R}{L}t} + c$$

$$i.e^{\frac{R}{L}t} = \frac{E}{R}\times e^{R/L\,t} + c \quad \text{---(iii)}$$

when t = 0 i = 0 we put in eqn(iii)

$$\Rightarrow 0 = \frac{E}{R}1 + c$$

$$\Rightarrow c = -E/R$$

$\Rightarrow i \cdot e^{\frac{R}{L}t} = \frac{E}{R} e^{\frac{R}{L}t} - \frac{E}{R}$

$\Rightarrow i = \frac{\frac{E}{R} e^{\frac{R}{L}t}}{e^{R/L\, t}} - \frac{E}{R} e^{-\frac{R}{L}t}$

$\Rightarrow i = \frac{E}{R} - \frac{E}{R} e^{-\frac{R}{L}t}$

$= \frac{E}{R}(1 - e^{-R/L\, t})$

$* \int e^{ax} \sin bx\, dx$

$= \frac{e^{ax}}{a^2+b^2}[a \sin bx - b \cos bx]$

$= \frac{e^{ax}}{a^2+b^2}[a \cos bx + b \sin bx]$

$\int e^{ax} \cos bx\, dx$

Q. So that the differenentral eqn fore the current is a electric circuit contarng an ~~cono~~ inducta (L) and the resistance (R) is series and attach by emf E sin wt satify the equation

$L \frac{di}{dt} + Ri = E \sin wt$ find the value of current at any time t if intially there is no current in the circuit $i = 0$ $t = 0$

Ans :- $L \frac{di}{dt} + Ri = E \sin wt$ —(1)

Reducing eqn (1) we have divided L both side.

$\frac{di}{dt} = \frac{R}{L} i = \frac{E \sin wt}{L}$

which is linear i

Here $P(t) = R/L$

$Q(t) = \frac{E \sin wt}{L}$

$I.F = e^{\int P(t)\, d(t)}$

$= e^{\int R/L\, dt}$

$= e^{R/L\, t}$

The required soln is

$i \cdot I.F = \int Q(t)$

$i \cdot e^{R/L t} = \int \frac{E \sin wt}{L} \cdot e^{R/L t} dt + C$

i.e $e^{R/L\,t} = \int \frac{E\sin\omega t}{L} \cdot e^{R/L\,t}\,dt + C$

$= \frac{E}{L}\int e^{\frac{R}{L}t}\sin\omega t\,dt + C$

$= \frac{E}{L}\;\frac{e^{\frac{R}{L}t}}{\left(\frac{R}{L}\right)^2+\omega^2}\left[\frac{R}{L}\sin\omega t - \omega\cos\omega t\right] + C$

$= \frac{E}{L}\;\frac{e^{\frac{R}{L}t}}{\frac{R^2+\omega^2 L^2}{L^2}}\left[\frac{R}{L}\sin\omega t - \omega\cos\omega t\right] + C$

$= \frac{E}{L}\cdot\frac{L^2 e^{R/L\,t}}{R^2+\omega^2L^2}\left[\frac{R}{L}\sin\omega t - \omega\cos\omega t\right] + C$

$= EL\;\frac{e^{R/L\,t}}{R^2+\omega^2L^2}\left[\frac{R}{L}\sin\omega t - \omega\cos\omega t\right] + C$

$\therefore i = \frac{EL}{R^2+\omega^2L^2}\left[\frac{R}{L}\sin\omega t - \omega\cos\omega t\right] + Ce^{-\frac{R}{L}t}$

If $t=0$, $i=0$

$\Rightarrow 0 = \frac{EL}{R^2+\omega^2L^2}[0-\omega] + C$

$\Rightarrow 0 = -\frac{\omega EL}{R^2+\omega^2} + C$

$\Rightarrow C = \frac{\omega EL}{R^2+\omega^2}$

$i = \frac{EL}{R^2+\omega^2}\left[\frac{R}{L}\sin\omega t - \omega\cos\omega t\right] + \frac{\omega EL}{R^2+\omega^2}e^{-\frac{R}{L}t}$

Q. Solve the equation $L\frac{di}{dt} + Ri = E_0 \sin\omega t$ where L, R and (E_0) = constant and discuss the case when t increases definitly. ($t = \infty$)

Q. $x\frac{dy}{dx} + y = x^4y^3$

Q. $x\frac{dy}{dx} + y\log y = xye^x$

Q. $\cancel{(y+1)}\cdot\frac{dy}{dx}(x^2y^3 + xy) = 1$

Q. $x\frac{dy}{dx} + y = x^4y^3$

Ans:- $x\frac{dy}{dx} + y = x^4y^3$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = x^3y^3$ (x devided by both side)

$\Rightarrow \frac{1}{y^3}\cdot\frac{dy}{dx} + \frac{y^{-2}}{x} = x^3$

Let $t = y^{-2}$

$\frac{dt}{dx} = -2y^3$

$\Rightarrow -\frac{1}{2}\frac{dt}{dx} + \frac{t}{x} = x^3$

$\Rightarrow \frac{dt}{dx} - \frac{dx}{2x} = -\frac{x^3}{2}$

$\therefore P(x) = -\frac{1}{2}x \quad Q(x) = -\frac{x^3}{2}$

$I.F = e^{\int P(x)\,dx}$

$= e^{\int -\frac{1}{2}dx}$

$= e^{\ln\frac{1}{\sqrt{2}}}$

$= \frac{1}{\sqrt{2}}$

General soln:-

$$u.\left(\frac{1}{\sqrt{x}}\right) = \int -\frac{x^3}{2}\cdot\frac{1}{\sqrt{x}}\,dx$$

$$= -\frac{1}{2}\int x^3 \cdot x^{-1/2}\,dx$$

$$= -\frac{1}{2}\int x^{5/2}\,dx$$

$$= -\frac{1}{2}\,\frac{x^{7/2}}{7/2} + c$$

$$\Rightarrow t\left(\frac{1}{\sqrt{x}}\right) = -\frac{x^{7/2}}{7/2} + c$$

$$\Rightarrow t = -\frac{x^{7/2}\sqrt{x}}{7} + \sqrt{x}\,c$$

$$\Rightarrow t = -\frac{x^4}{7} + \sqrt{x}\,c$$

$$\Rightarrow \frac{1}{y^2} = -\frac{x^4}{7} + \sqrt{x}\,c$$

Q2. $x\frac{dy}{dx} + y\log y = xye^x$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x}\log y = ye^x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} + \frac{\log y}{x} = e^x$$

Let $t = \log y$

$$\Rightarrow \frac{dt}{dx} = \frac{1}{y}\frac{dy}{dx}$$

$$\therefore \frac{dt}{dx} + \frac{t}{x} = e^x$$

$P(x) = 1/x \quad Q(x) = e^x$

$$I.F = e^{\int P(x)\,dx}$$

$$= e^{\int 1/x\,dx}$$

$$= e^{\ln x}$$

$$= x$$

General soln:-

$$t \cdot x = \int x \cdot e^x dx$$

$$= x \cdot e^x - \int e^x dx$$

$$= xe^x - e^x + c$$

$$\Rightarrow t \cdot x = e^x(x-1) + c$$

$$\Rightarrow \frac{1}{\log y} = \frac{e^x(x-1)}{x} + \frac{c}{x}$$

$$\Rightarrow \frac{1}{\log y} = e^x\left(1 - \frac{1}{x}\right) + \frac{c}{x}$$

$$\Rightarrow \log y = \frac{1-x}{e^x} + \frac{x}{c}$$

$$\Rightarrow y = e^{\frac{(1-x)}{e^x}} \cdot e^{\frac{x}{c}}$$

Q $\frac{dy}{dx}(x^2y^3 + xy) = 1$

$$\Rightarrow x^2y^3 + xy = \frac{dy}{dx}$$

$$\Rightarrow \frac{dx}{dy} - xy = x^2y^3$$

$$\Rightarrow \frac{1}{x}\frac{dx}{dy} - \frac{xy}{x^2} = \frac{x^2y^3}{x^2}$$

$$\Rightarrow \frac{1}{x^2}\frac{dx}{dy} - \frac{xy}{x^2} = \frac{x^2y^3}{x^2}$$

$$\Rightarrow \frac{1}{x^2}\frac{dx}{dy} - \frac{y}{x} = y^3$$

Let $u = \frac{1}{x}$

$$\Rightarrow \frac{du}{dy} = -x^{-2}\frac{dx}{dy}$$

$$\Rightarrow -\frac{du}{dy} - uy = y^3$$

$$\Rightarrow \frac{du}{dy} + uy = -y^3$$

Date- 5th december

(1) $(1+y)\cdot dx-(1-x)dy=0$

$\Rightarrow \cancel{\frac{dy}{dx}}$

$\Rightarrow (1+y)dx=(1-x)dy$

$\Rightarrow \frac{dy}{dx}=\frac{1-x}{1+y}$

$\Rightarrow (1+y)dy=(1-x)dx$

$\Rightarrow dy+y^2dy=x-x^2dx$

(2) $(x+2y+1)dx-(2x-3)dy=0$

$\Rightarrow (x+2y+1)dx=2x-3\,dy$

$\Rightarrow \frac{dy}{dx}=\frac{x+2y+1}{2x-3}$

Let $x=X+h$

$y=Y+k$

$\Rightarrow \frac{dy}{dx}=\frac{X+h+2(Y+k)+1}{2(X+h)-3}$

$=\frac{X+h+2Y+k+1}{2X+2h-3}$

$=\frac{x+h+2y+k+1}{2x+2h-3}$

$=\frac{x+2y+h+k+1}{2x+2h-3}$

$=\frac{(x+2y)+(h+k+1)}{2x+2h-3}$